About partonic intrinsic motion and SSA

Estimate of transverse motion of quarks

TMDs: spin-intrinsic motion correlations in distribution and fragmentation functions

Sivers and Collins functions; SSA in SIDIS

Coupling Collins function and Transversity

What do we learn from Sivers functions?

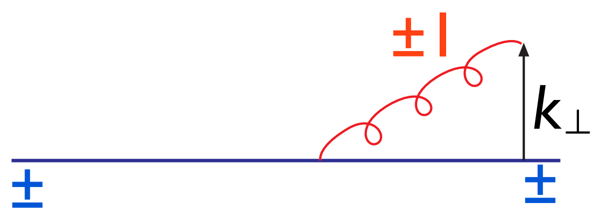
The full structure of TMDs in SIDIS

Partonic intrinsic motion

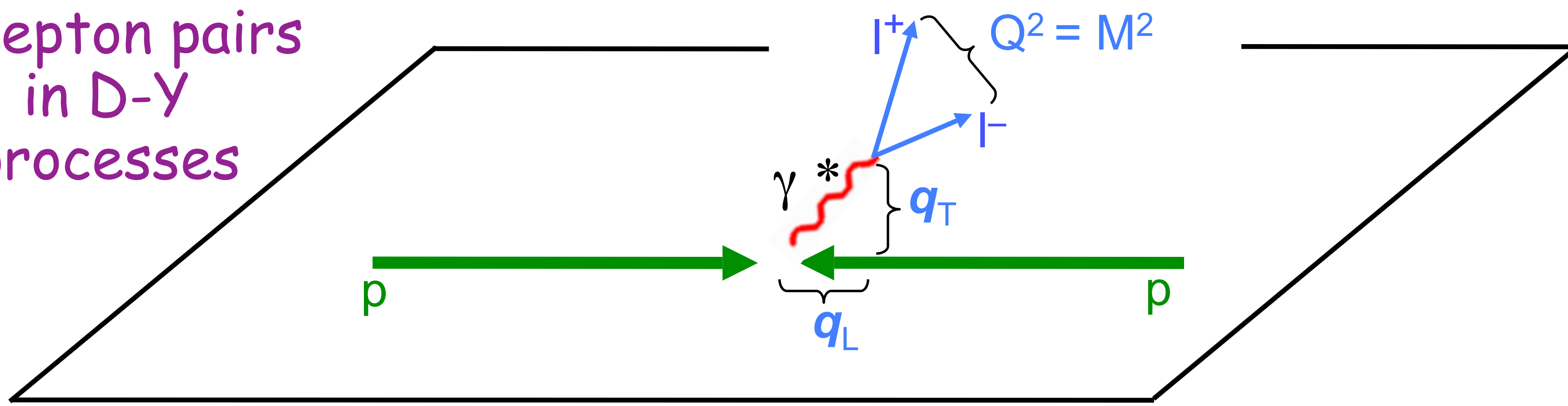
Plenty of theoretical and experimental evidence for transverse motion of partons within nucleons and of hadrons within fragmentation jets

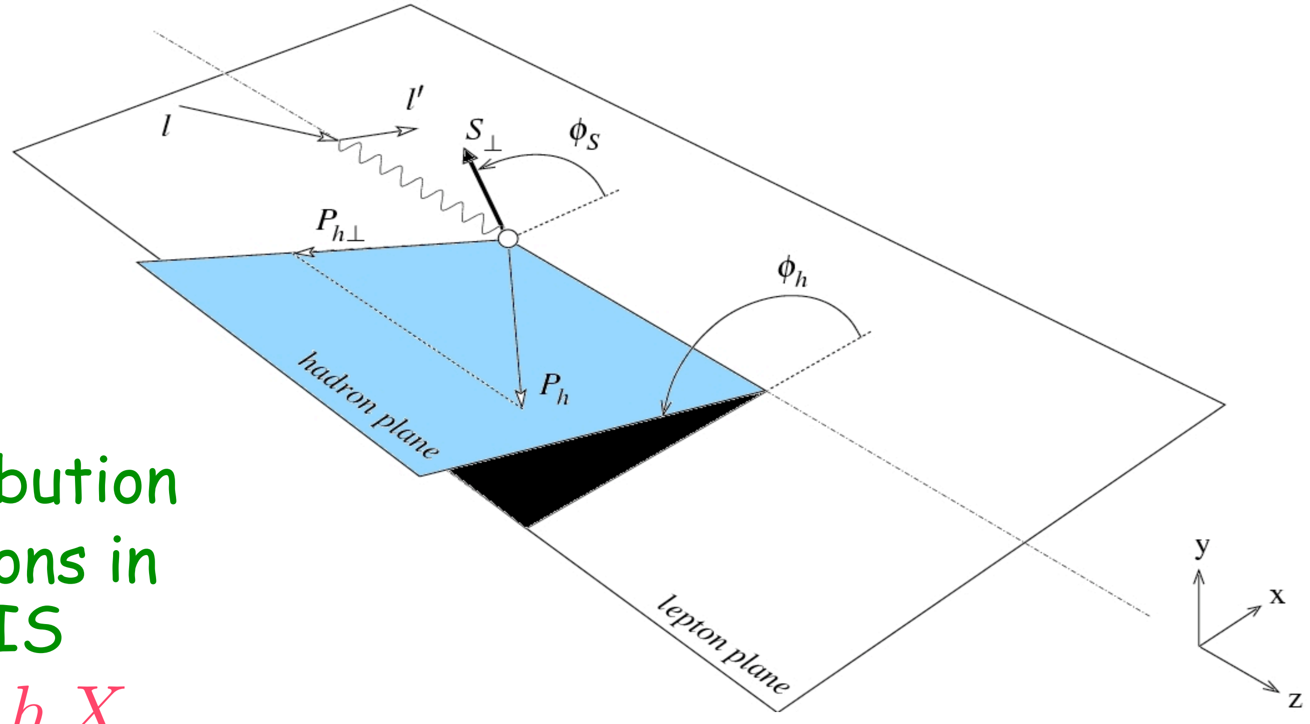
uncertainty principle $\Delta x \simeq 1 \text{ fm} \;\; \Rightarrow \;\; \Delta p \simeq 0.2 \text{ GeV/c}$

gluon radiation



q_T distribution of lepton pairs in D-Y processes

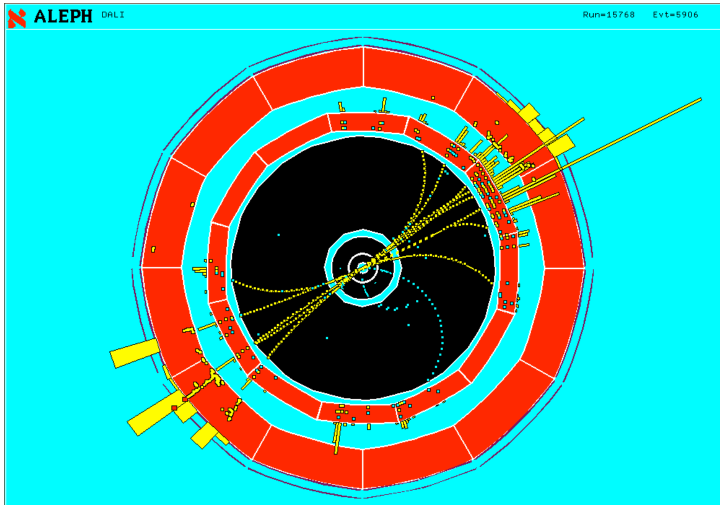




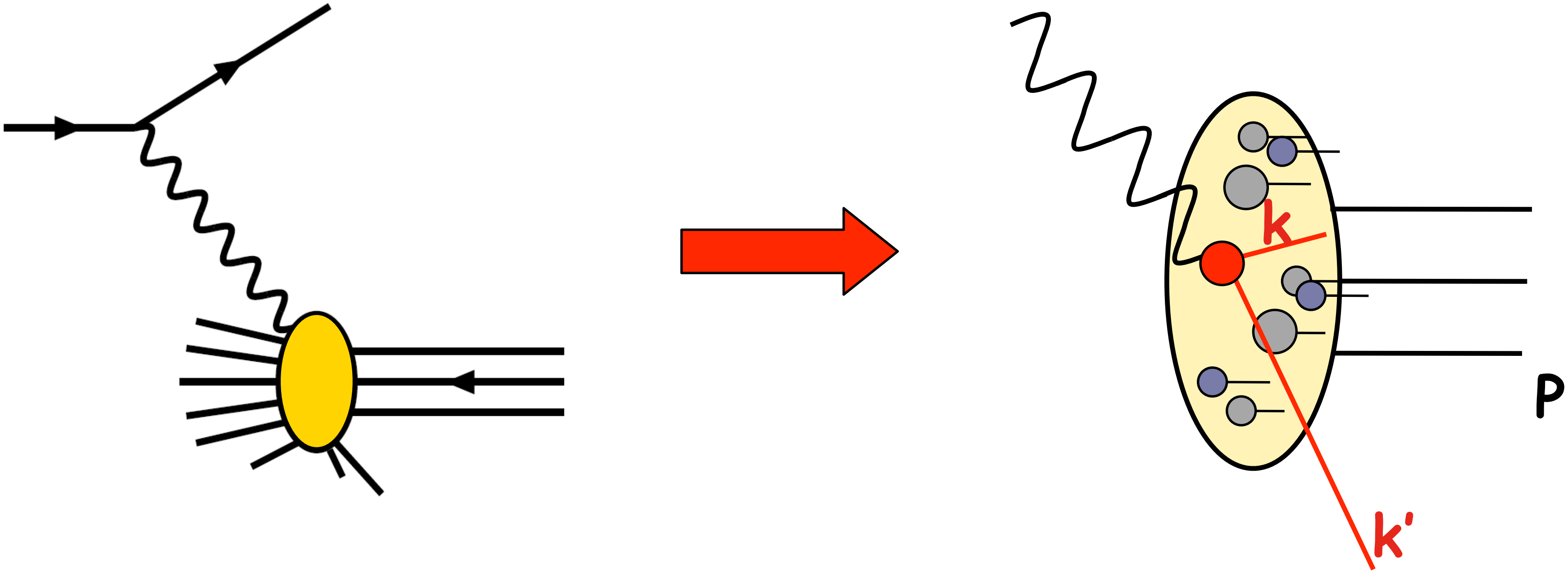
p_T distribution of hadrons in SIDIS

$\ell\, p \to \ell\, h\, X$

Hadron distribution in jets in e^+e^- processes



Parton Model with intrinsic motion



Assume: struck parton carries 4-momentum k

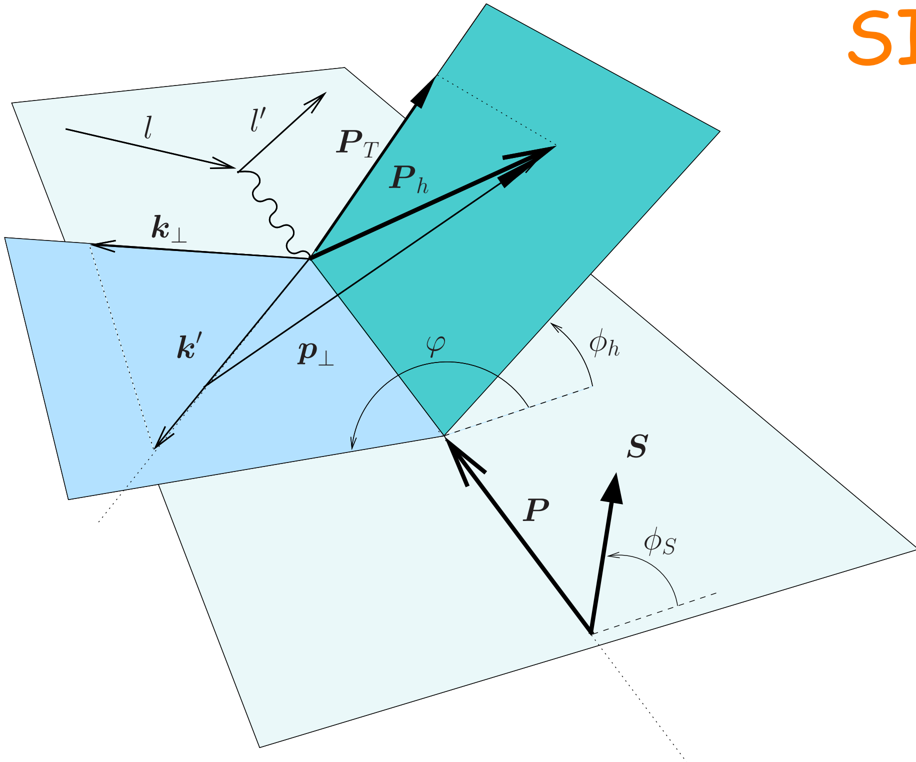
$$k = \left(xP_0 + \frac{k_\perp^2}{4xP_0},\ \boldsymbol{k}_\perp,\ -xP_0 + \frac{k_\perp^2}{4xP_0} \right) \qquad k' = k + q$$

$$k^2 = 0 \qquad \boldsymbol{k}_\perp = k_\perp(\cos\varphi,\ \sin\varphi,\ 0) \qquad x = k^- / P^-$$

$$P = (P_0,\ 0,\ 0,\ -P_0)$$

SIDIS in parton model with intrinsic $k_\perp$

observables at $\mathcal{O}\left(\frac{k_\perp}{Q}\right)$



$$
\begin{aligned}
z_h &= \frac{P \cdot P_h}{P \cdot q} = z \\
\boldsymbol{P}_T &= z\,\boldsymbol{k}_\perp + \boldsymbol{p}_\perp \\
x_B &= \frac{Q^2}{2P \cdot q} = x \\
y &= \frac{P \cdot q}{P \cdot \ell} = \frac{Q^2}{xs}
\end{aligned}
$$

factorization holds at large Q^2, and $P_T \approx k_\perp \approx \Lambda_{\rm QCD}$

(Ji, J.P. Ma, Yuan)

$$
\mathrm{d}\sigma^{\ell p \to \ell h X} = \sum_q f_q(x, \boldsymbol{k}_\perp; Q^2) \otimes \mathrm{d}\hat{\sigma}^{\ell q \to \ell q}(y, \boldsymbol{k}_\perp; Q^2) \otimes D_q^h(z, \boldsymbol{p}_\perp; Q^2)
$$

Elementary Mandelstam variables:

$$\hat{s} = (\ell + k)^2 \qquad \hat{t} = (\ell - \ell')^2 \qquad \hat{u} = (k - \ell')^2 \qquad \ell = E(1,\, \boldsymbol{\ell})$$

$$\hat{s} \;=\; xs - 2\,\boldsymbol{\ell}\cdot\boldsymbol{k}_\perp - k_\perp^2\,\frac{x_{_B}}{x}\left(1 - \frac{x_{_B}\,s}{Q^2}\right)$$

$$\hat{u} \;=\; -x\left(s - \frac{Q^2}{x_{_B}}\right) + 2\,\boldsymbol{\ell}\cdot\boldsymbol{k}_\perp - k_\perp^2\,\frac{x_{_B}\,s}{x\,Q^2}$$

The on shell condition for the final quark

$$k'^2 = 2q\cdot k - Q^2 = \hat{s} + \hat{t} + \hat{u} = 0$$

implies

$$x = \frac{1}{2}\,x_{_B}\left(1 + \sqrt{1 + \frac{4k_\perp^2}{Q^2}}\right)$$

neglecting $\mathcal{O}(k_\perp^2/Q^2)$ terms one has

$$\hat{s} = (\ell + k)^2 = sx\left[1 - \frac{2k_\perp}{Q}\sqrt{1-y}\,\cos\varphi\right]$$

$$\hat{u} = (\ell - k')^2 = -sx(1-y)\left[1 - \frac{2k_\perp}{Q\sqrt{1-y}}\,\cos\varphi\right]$$

$$x = x_{_B} \qquad z = z_h \qquad \boldsymbol{P}_T = z\,\boldsymbol{k}_\perp + \boldsymbol{p}_\perp$$

"Cahn effect"

$$d\hat{\sigma}^{\ell q \to \ell q} \propto \hat{s}^2 + \hat{u}^2 = \frac{Q^4}{y^2}\left[1 + (1-y)^2 - 4\,\frac{k_\perp}{Q}\,(2-y)\,\sqrt{1-y}\,\cos\varphi\right]$$

$$\frac{\mathrm{d}\sigma^{\ell p \to \ell h X}}{\mathrm{d}\Phi_h} \propto A + B\cos\Phi_h$$

assuming: $$\begin{cases} f_{q/p}(x,k_\perp) = f_q(x)\,\dfrac{1}{\pi\langle k_\perp^2\rangle}\,e^{-k_\perp^2/\langle k_\perp^2\rangle} \\[2ex] D_q^h(z,p_\perp) = D_q^h(z)\,\dfrac{1}{\pi\langle p_\perp^2\rangle}\,e^{-p_\perp^2/\langle p_\perp^2\rangle} \end{cases}$$

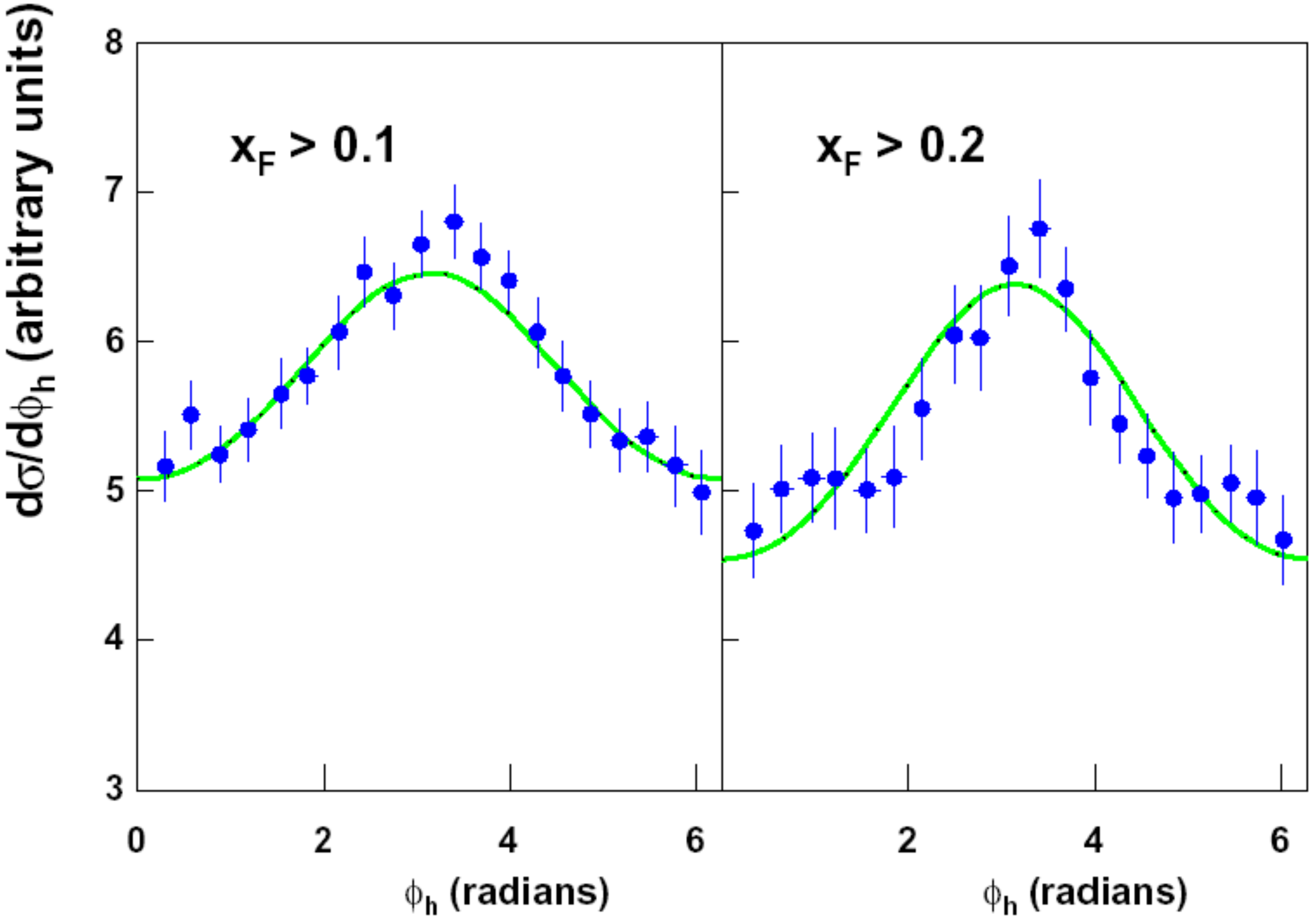
one finds:

$$\frac{d^5\sigma^{\ell p\to\ell h X}}{dx_B\,dQ^2\,dz_h\,d^2\boldsymbol{P}_T} \simeq \sum_q \frac{2\pi\alpha^2 e_q^2}{Q^4}\,f_q(x_B)\,D_q^h(z_h)\Big[1+(1-y)^2 -4\,\frac{(2-y)\sqrt{1-y}\,\langle k_\perp^2\rangle\,z_h\,P_T}{\langle P_T^2\rangle\,Q}\,\cos\phi_h\Big]\frac{1}{\pi\langle P_T^2\rangle}\,e^{-P_T^2/\langle P_T^2\rangle}$$

with $\langle P_T^2\rangle = \langle p_\perp^2\rangle + z_h\langle k_\perp^2\rangle$ ⇒

clear dependence on $\langle p_\perp^2\rangle$ and $\langle k_\perp^2\rangle$ (assumed to be constant)

Find best values by fitting data on Φ_h and P_T dependences

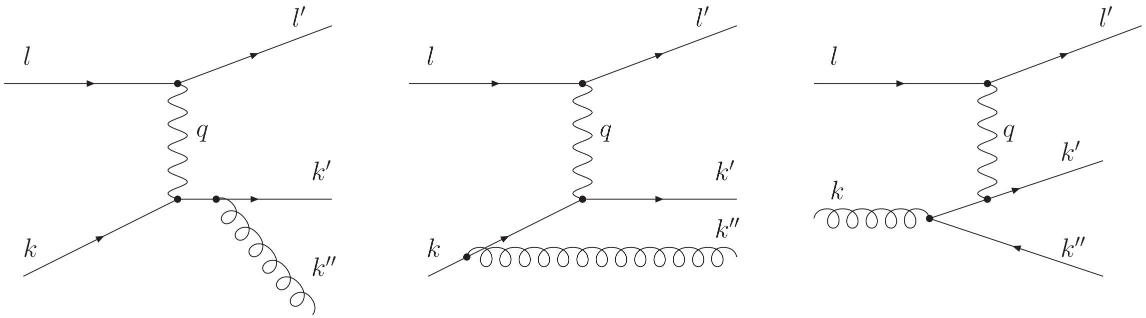


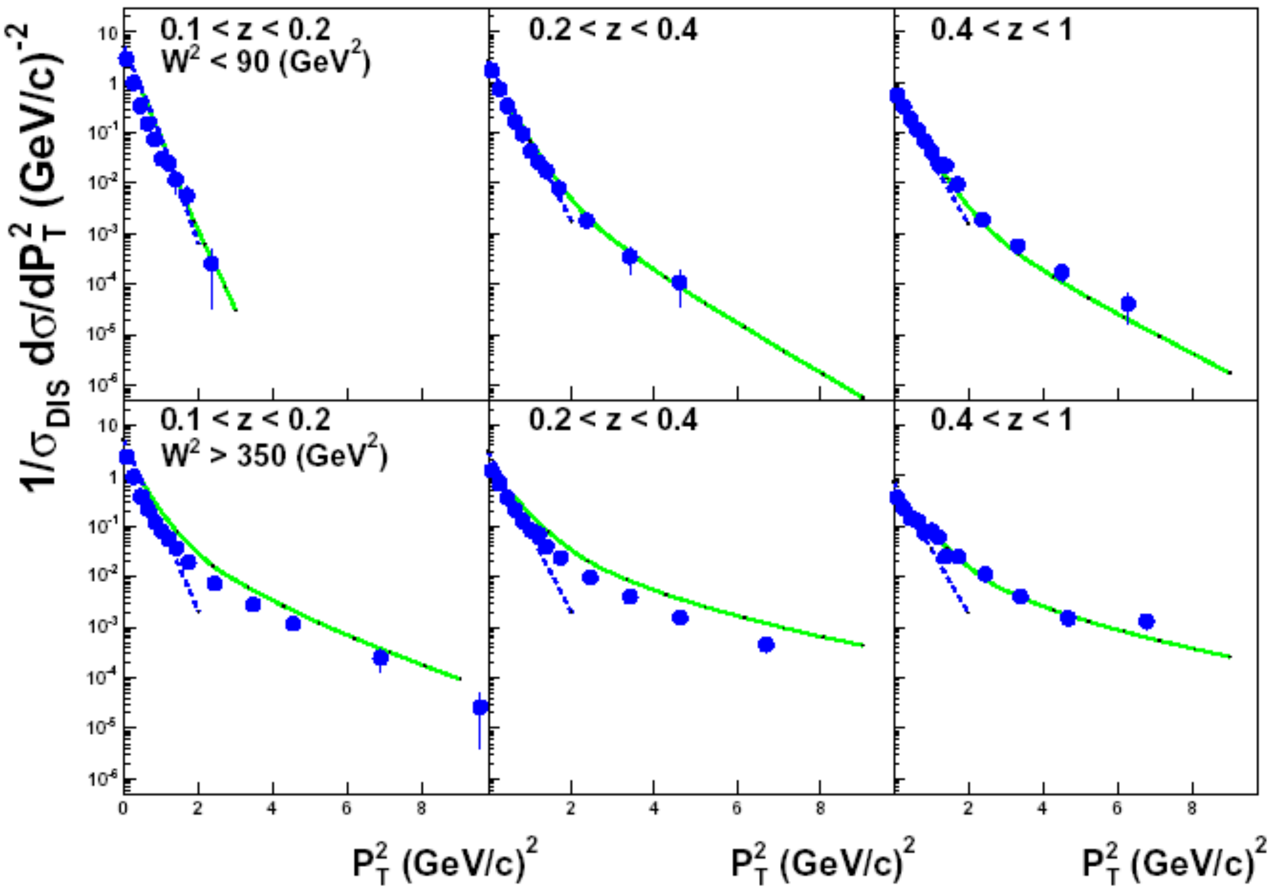
EMC data, μp and μd, *E* between 100 and 280 GeV

$$\langle k_\perp^2 \rangle = 0.28\ (\mathrm{GeV})^2 \qquad \langle p_\perp^2 \rangle = 0.25\ (\mathrm{GeV})^2$$

M.A., M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia and A. Prokudin, C. Türk

Large P_T data explained by NLO QCD corrections

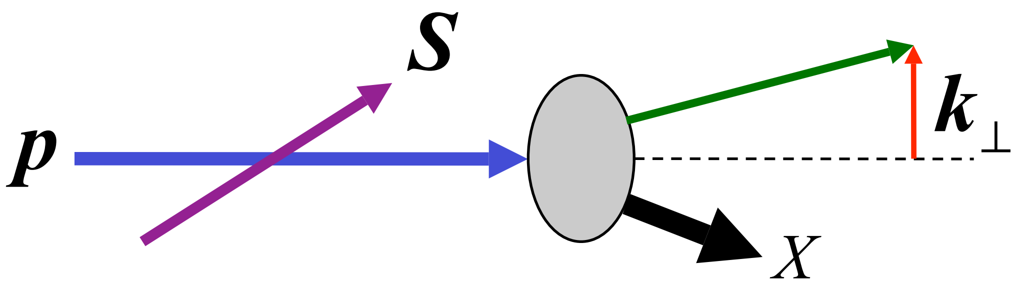




EMC data

How does intrinsic motion help with SSA?

One can introduce spin-$\mathbf{k}_\perp$ correlation in the Parton Distribution Functions (PDFs) and in the parton Fragmentation Functions (FFs)



Only possible (scalar) correlation is

$$\boldsymbol{S} \cdot (\boldsymbol{p} \times \boldsymbol{k}_\perp)$$

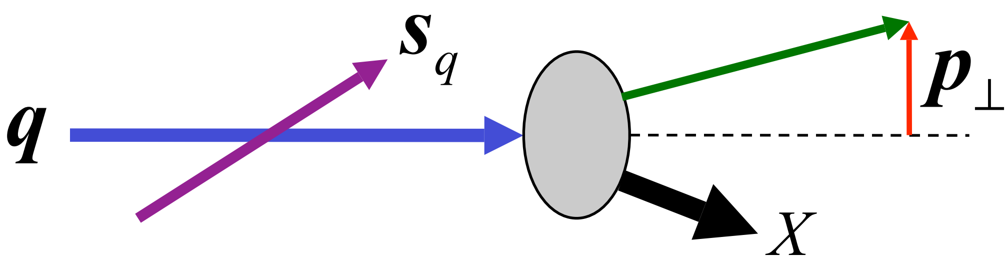
TMDs: Sivers function

$$\begin{aligned} f_{q/p,\boldsymbol{S}}(x,\boldsymbol{k}_\perp) &= f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp) \\ &= f_{q/p}(x,k_\perp) - \frac{k_\perp}{M} f_{1T}^{\perp q}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp) \end{aligned}$$

Boer-Mulders function

$$\begin{aligned} f_{q,\boldsymbol{s}_q/p}(x,\boldsymbol{k}_\perp) &= \frac{1}{2} f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q^\uparrow/p}(x,k_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp) \\ &= \frac{1}{2} f_{q/p}(x,k_\perp) - \frac{k_\perp}{2M} h_1^{\perp q}(x,k_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp) \end{aligned}$$

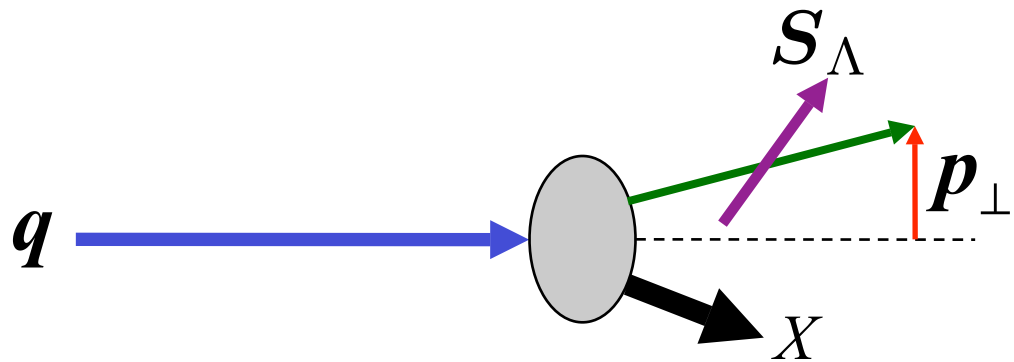
Spin-$\mathbf{p}_\perp$ correlations in fragmentation process (case of final spinless hadron)



Collins function

$$
\begin{aligned}
D_{h/q,\boldsymbol{s}_q}(z,\boldsymbol{p}_\perp) &= D_{h/q}(z,p_\perp) + \frac{1}{2}\Delta^N D_{h/q^\uparrow}(z,p_\perp)\, \boldsymbol{s}_q \cdot (\hat{\boldsymbol{p}}_q \times \hat{\boldsymbol{p}}_\perp) \\
&= D_{h/q}(z,p_\perp) + \frac{p_\perp}{zM_h} H_1^{\perp q}(z,p_\perp)\, \boldsymbol{s}_q \cdot (\hat{\boldsymbol{p}}_q \times \hat{\boldsymbol{p}}_\perp)
\end{aligned}
$$

Spin-$p_\perp$ correlations in fragmentation process (case of final spin 1/2 hadron)



polarizing fragmentation function

$$
\begin{aligned}
D_{\Lambda,\boldsymbol{S}_\Lambda/q}(z,\boldsymbol{p}_\perp) &= \frac{1}{2}\,D_{\Lambda/q}(z,p_\perp) + \frac{1}{2}\,\Delta^N D_{\Lambda^\uparrow/q}(z,p_\perp)\,\boldsymbol{S}_\Lambda\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp) \\
&= \frac{1}{2}\,D_{\Lambda/q}(z,p_\perp) + \frac{p_\perp}{z\,M_\Lambda}\,D_{1T}^{\perp q}(z,p_\perp)\,\boldsymbol{S}_\Lambda\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp)
\end{aligned}
$$

Sivers effect in SIDIS

$$\mathrm{d}\sigma^{\uparrow,\downarrow} = \sum_q f_{q/p^{\uparrow,\downarrow}}(x, \boldsymbol{k}_\perp; Q^2) \otimes \mathrm{d}\hat{\sigma}(y, \boldsymbol{k}_\perp; Q^2) \otimes D_{h/q}(z, \boldsymbol{p}_\perp; Q^2)$$

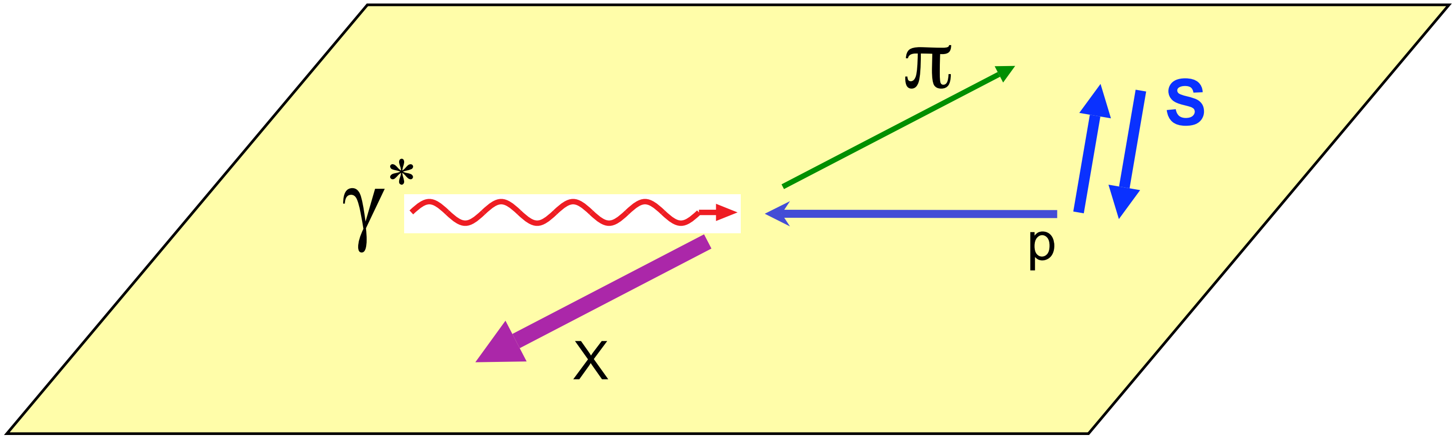
$$f_{q/p^{\uparrow,\downarrow}}(x, \boldsymbol{k}_\perp) = f_{q/p}(x, k_\perp) \pm \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x, k_\perp)\, \boldsymbol{S} \cdot (\hat{\boldsymbol{p}} \times \hat{\boldsymbol{k}}_\perp)$$

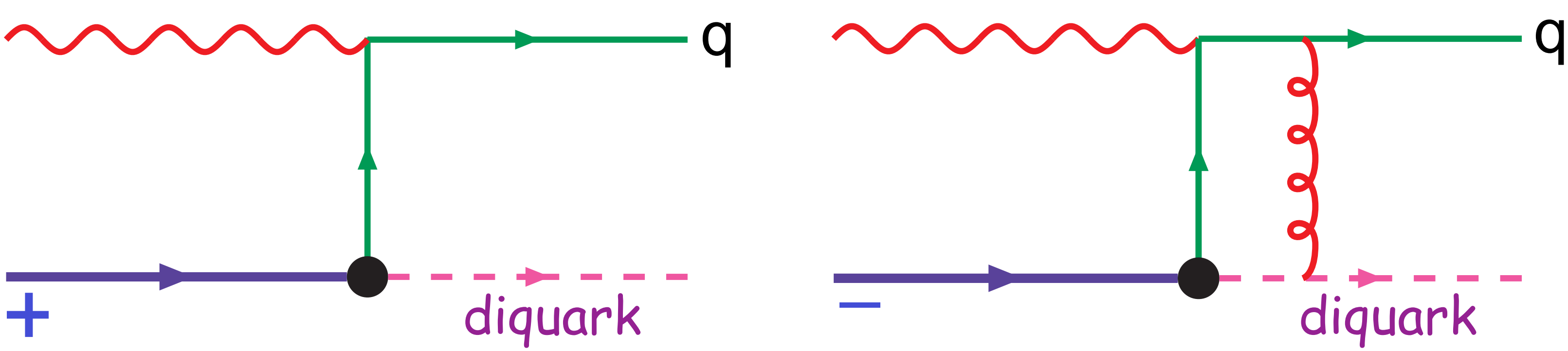
$$\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow =$$

$$\sum_q \Delta^N f_{q/p^\uparrow}(x, k_\perp)\, \underbrace{\boldsymbol{S} \cdot (\hat{\boldsymbol{p}} \times \boldsymbol{k}_\perp)}_{\sin(\varphi - \Phi_S)} \otimes \mathrm{d}\hat{\sigma}(y, \boldsymbol{k}_\perp) \otimes D_{h/q}(z, \boldsymbol{p}_\perp)$$

$$\Delta^N f_{q/p^\uparrow} = -\frac{2k_\perp}{M}\, f_{1T}^{\perp q}$$

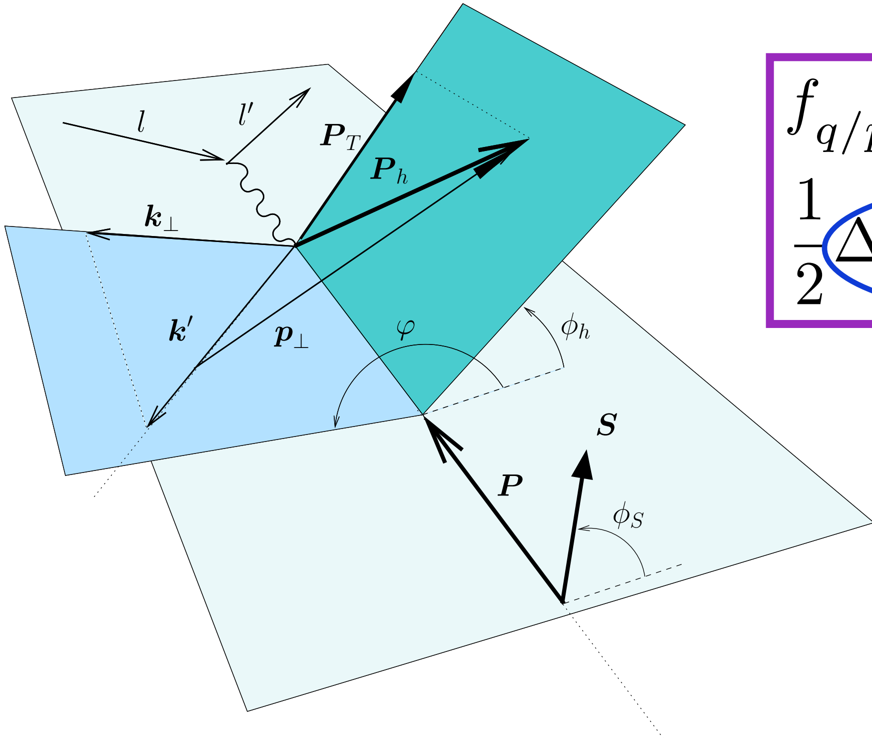
Brodsky, Hwang, Schmidt model for Sivers function





$$\boldsymbol{S} \cdot (\boldsymbol{p} \times \boldsymbol{P}_T) \propto P_T \sin(\Phi_\pi - \Phi_S)$$

needs $\mathbf{k}_\perp$ dependent quark distribution in $p^\uparrow$:
Sivers function



$$f_{q/p,\boldsymbol{S}}(x,\boldsymbol{k}_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

$$\boldsymbol{p}_\perp = \boldsymbol{P}_T - z\,\boldsymbol{k}_\perp$$

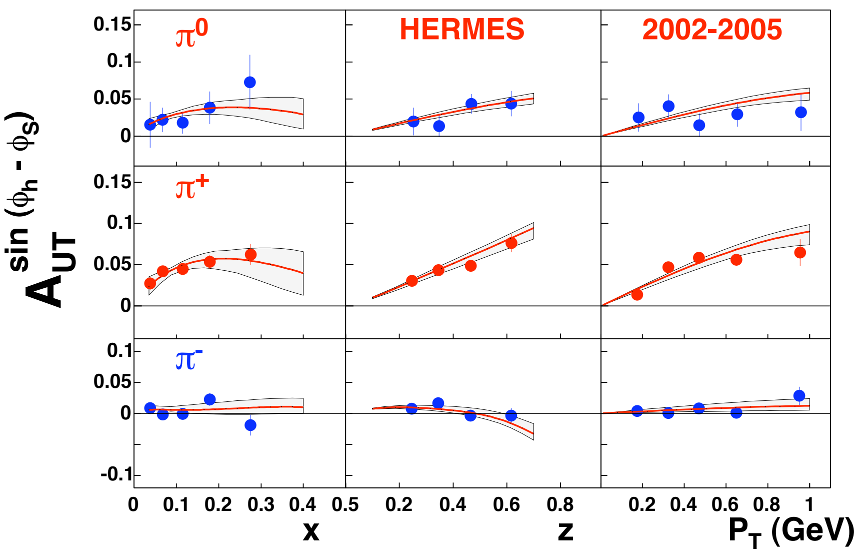
$$A_{UT}^{\sin(\Phi_h-\Phi_S)} \equiv 2\,\frac{\int d\Phi_S\,d\Phi_h\,[d\sigma^\uparrow - d\sigma^\downarrow]\,\sin(\Phi_h-\Phi_S)}{\int d\Phi_S\,d\Phi_h\,[d\sigma^\uparrow + d\sigma^\downarrow]}$$

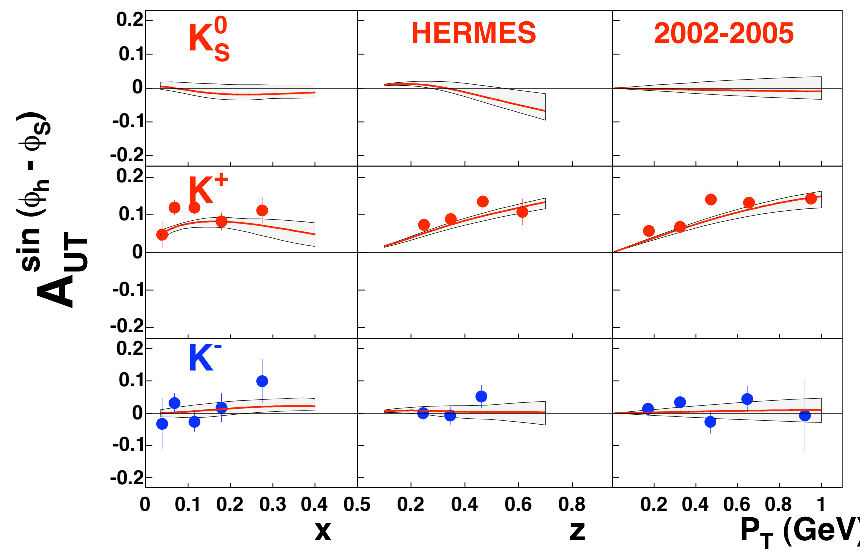
$$\frac{\sum_q \int d\Phi_S\,d\Phi_h\,d^2\boldsymbol{k}_\perp\,\Delta^N f_{q/p^\uparrow}(x,k_\perp)\,\sin(\varphi-\Phi_S)\,\frac{d\hat\sigma^{\ell q\to\ell q}}{dQ^2}\;D_{h/q}(z,p_\perp)\sin(\Phi_h-\Phi_S)}{\sum_q \int d\Phi_S\,d\Phi_h\,d^2\boldsymbol{k}_\perp\;f_{q/p}(x,k_\perp)\,\frac{d\hat\sigma^{\ell q\to\ell q}}{dQ^2}\;D_{h/q}(z,p_\perp)}$$

Fit of HERMES data on $A_{UT}^{\sin(\Phi_h-\Phi_S)}$

M. A., M. Boglione, U. D'Alesio, A. Kotzinian, S. Melis, F. Murgia,A. Prokudin and C. Türk

e-Print: arXiv:0805.2677

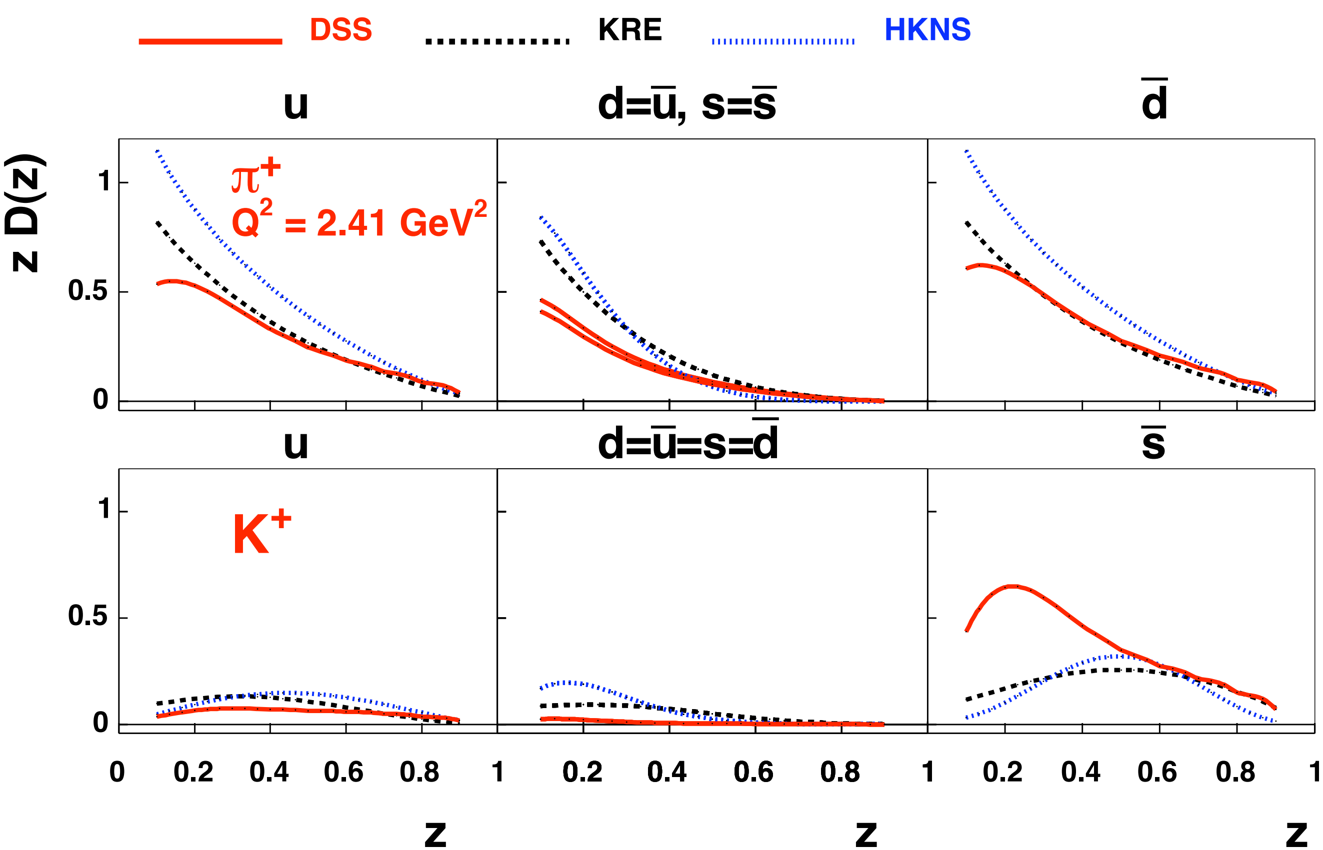




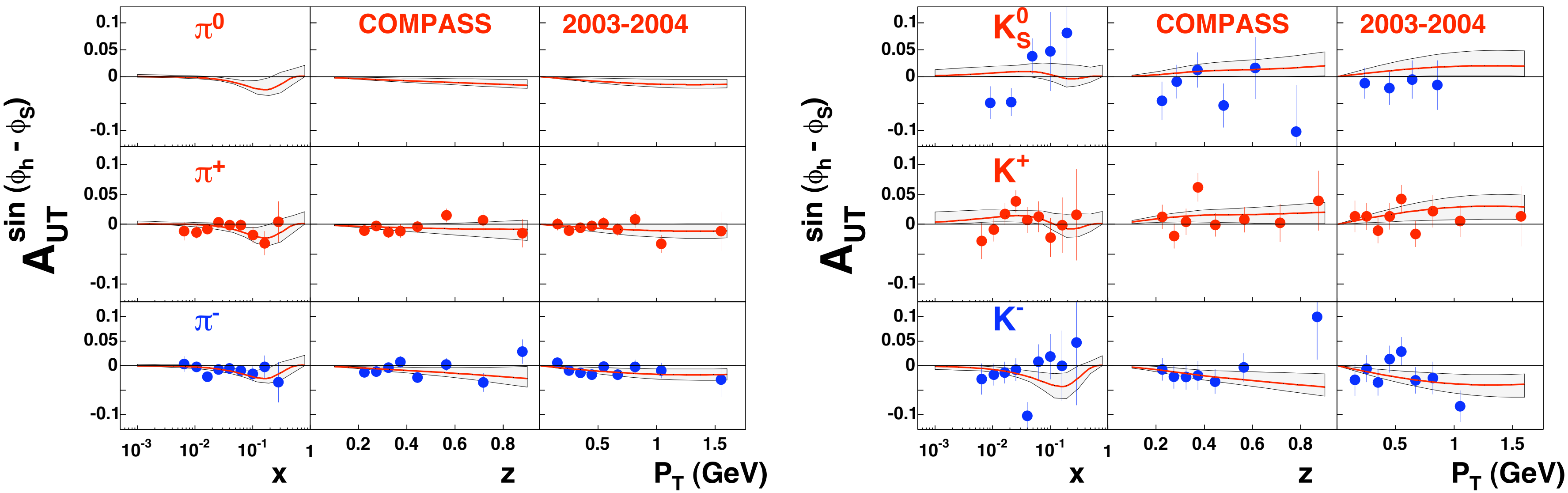
some problems with K^+ data

new set of fragmentation functions, based on pion and kaon production analysis

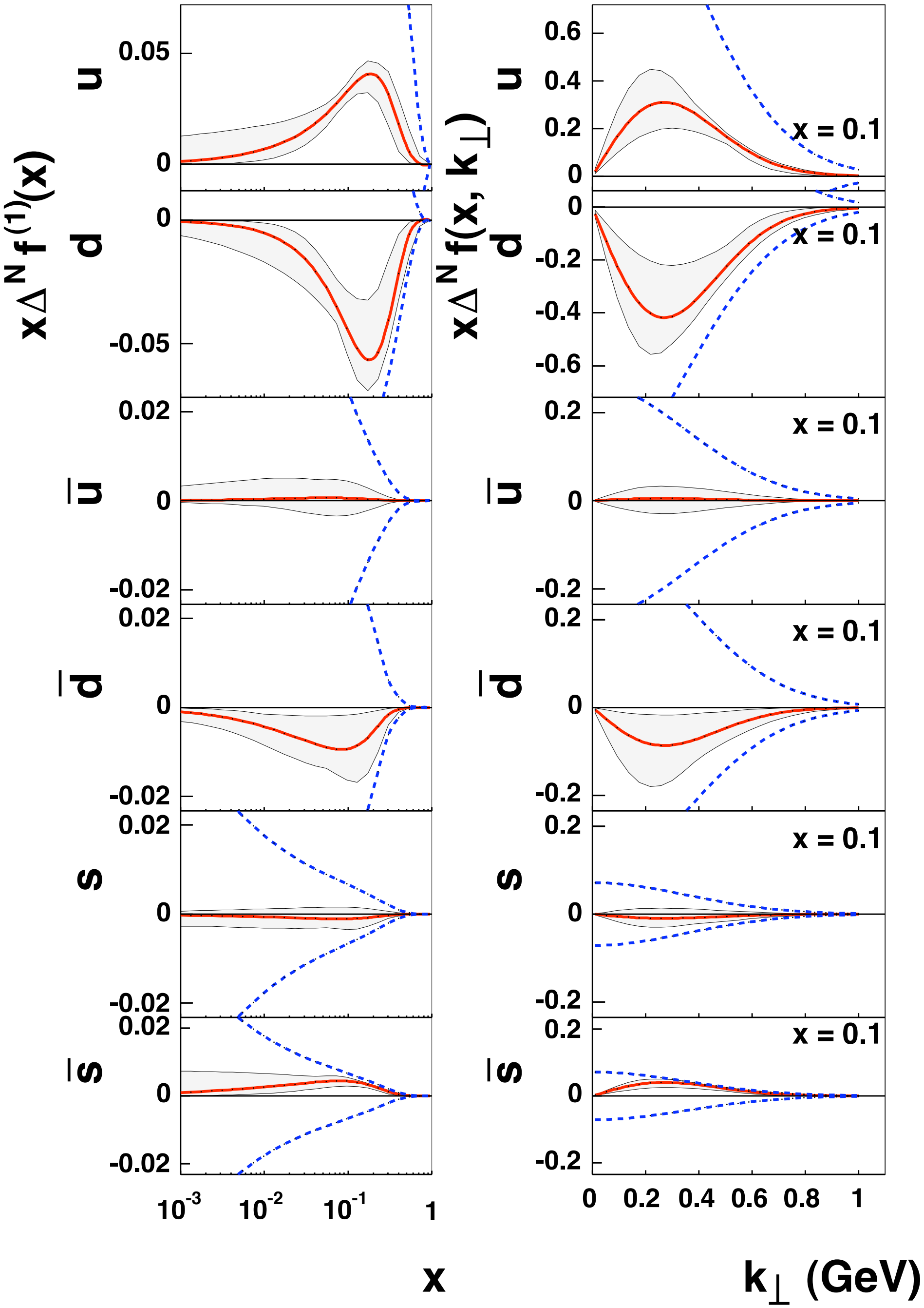
D. de Florian, R. Sassot, and M. Stratmann, Phys. Rev. D75, 114010 (2007)



Fit of COMPASS data on deuteron target



$$A_{UT}^{\sin(\Phi_h-\Phi_S)} \propto \underbrace{(\Delta^N f_{u/p^\uparrow} + \Delta^N f_{d/p^\uparrow})}_{\text{cancellation}}(4D_{h/u} + D_{h/d})$$



extracted Sivers functions

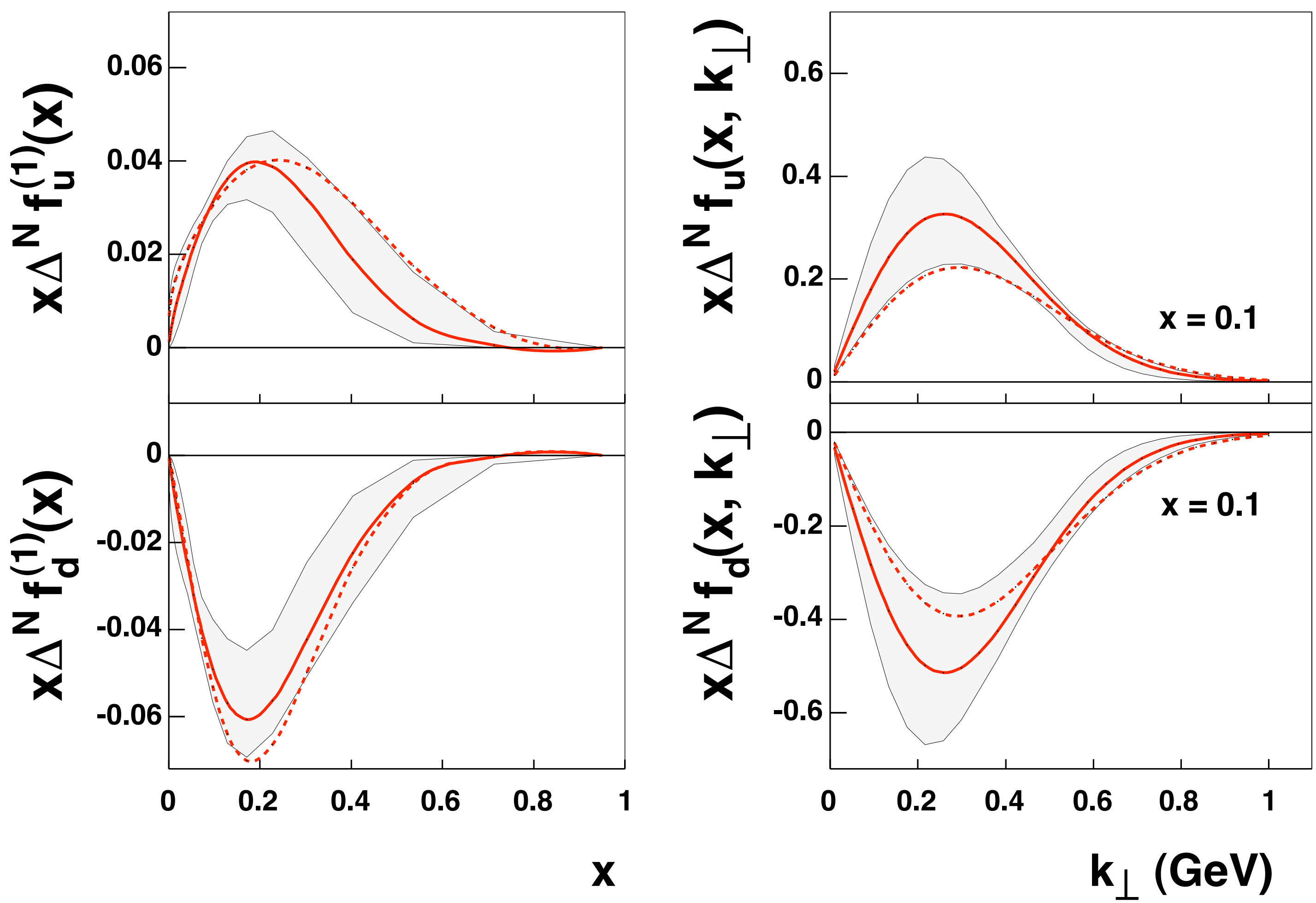
$$\Delta^N f_{u/p^\uparrow} > 0$$

$$\Delta^N f_{d/p^\uparrow} < 0$$

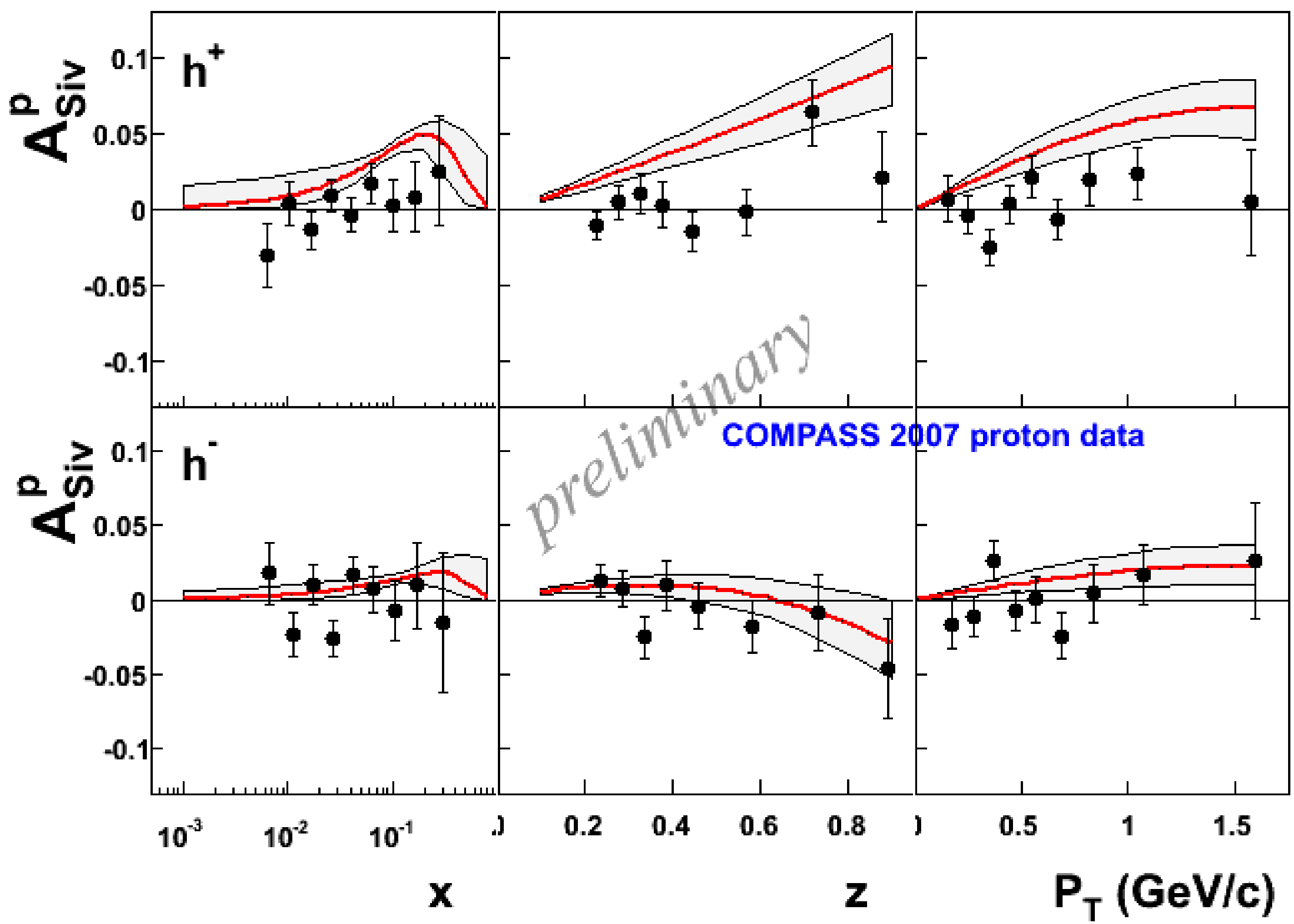
$$\Delta^N f_{\bar{s}/p^\uparrow} > 0$$

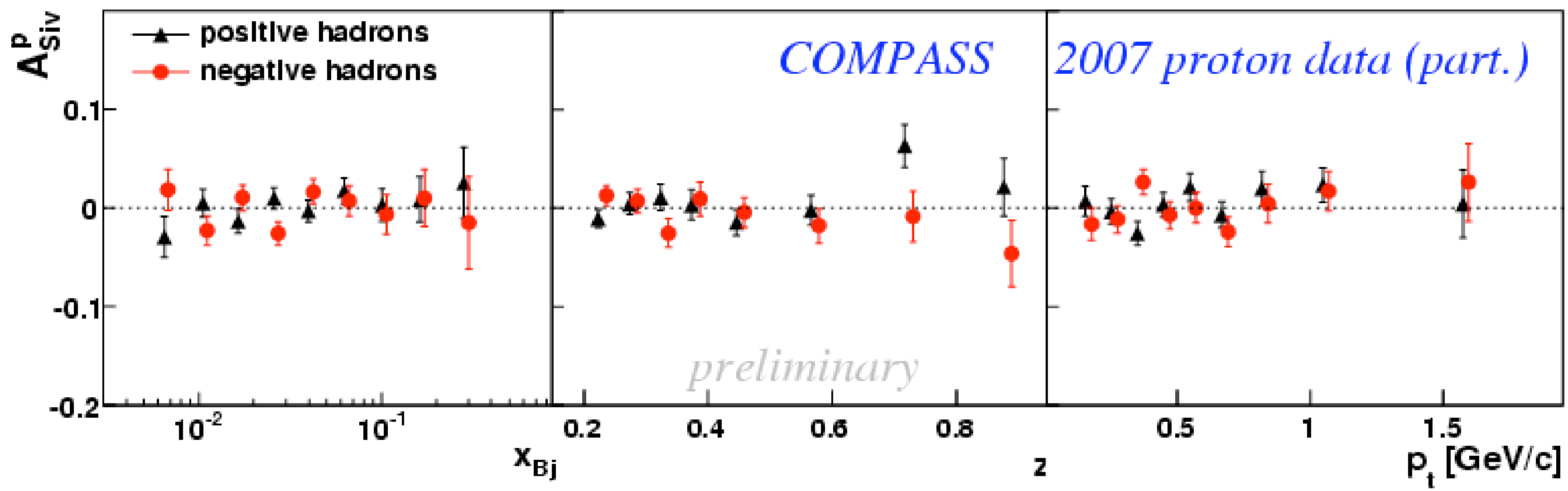
$$\Delta^N f^{(1)}_{q/p^\uparrow}(x) \equiv \int d^2 \boldsymbol{k}_\perp \frac{k_\perp}{4m_p} \Delta^N f_{q/p^\uparrow}(x, k_\perp) = -f_{1T}^{\perp(1)q}(x)$$

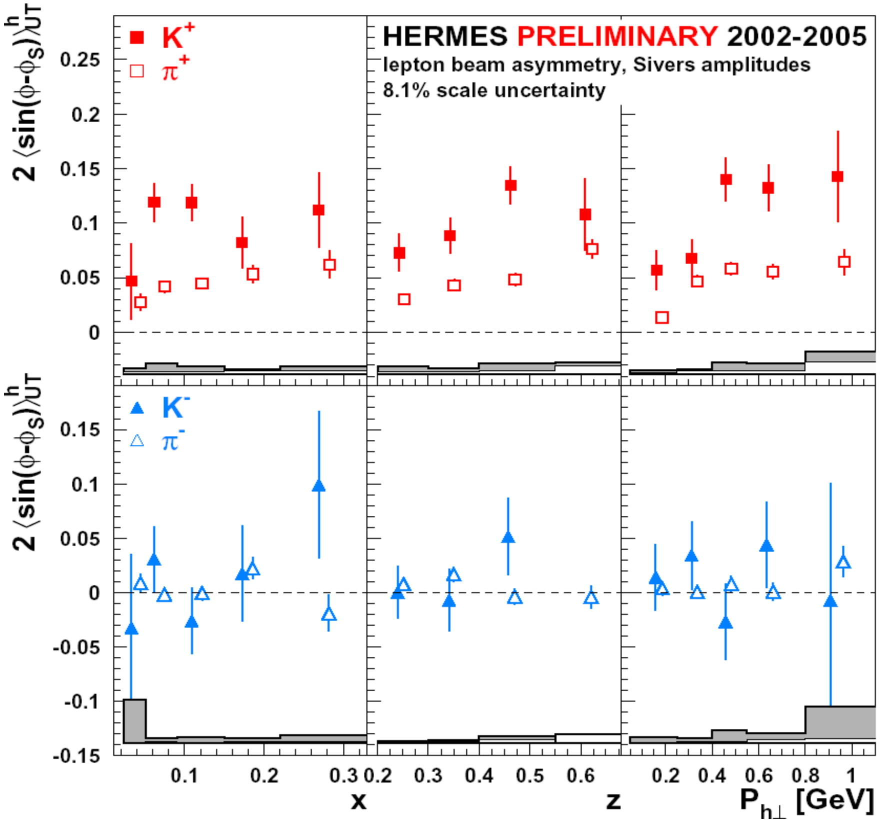
u and d Sivers functions rather well determined



Comparison of predictions with COMPASS data, proton target



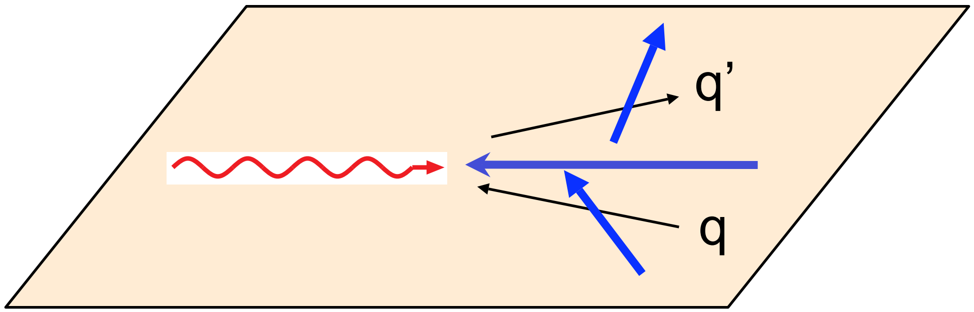




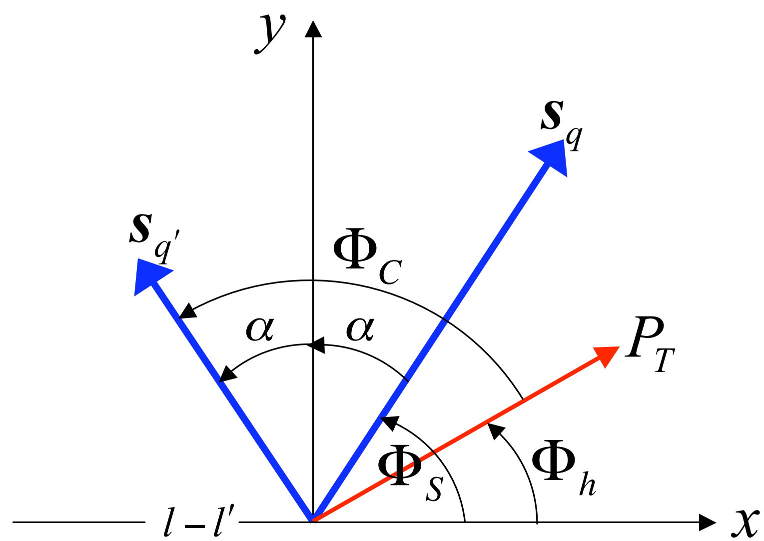
discrepancy between HERMES and COMPASS data on Sivers asymmetry?

Collins effect in SIDIS

Spin effect comes from fragmentation of a transversely polarized quark



initial q spin is transfered to final q', which fragments



$$D_{h/q,\boldsymbol{s}_q}(z,\boldsymbol{p}_\perp) = D_{h/p}(z,p_\perp) + \frac{1}{2}\Delta^N D_{h/q^\uparrow}(z,p_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp)$$

$$\boldsymbol{s}_{q'}\cdot(\hat{\boldsymbol{p}}_{q'}\times\hat{\boldsymbol{p}}_\perp) \simeq \sin(\Phi_h+\Phi_S)$$

$$\Delta^N D_{h/q^\uparrow}(z,\boldsymbol{p}_\perp) \equiv \Delta^N D_{h/q^\uparrow}(z,p_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp)$$

$$\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow} = \sum_q h_{1q}(x, k_\perp) \otimes \mathrm{d}\Delta\hat{\sigma}(y, \boldsymbol{k}_\perp) \otimes \Delta^N D_{h/q^\uparrow}(z, \boldsymbol{p}_\perp)$$

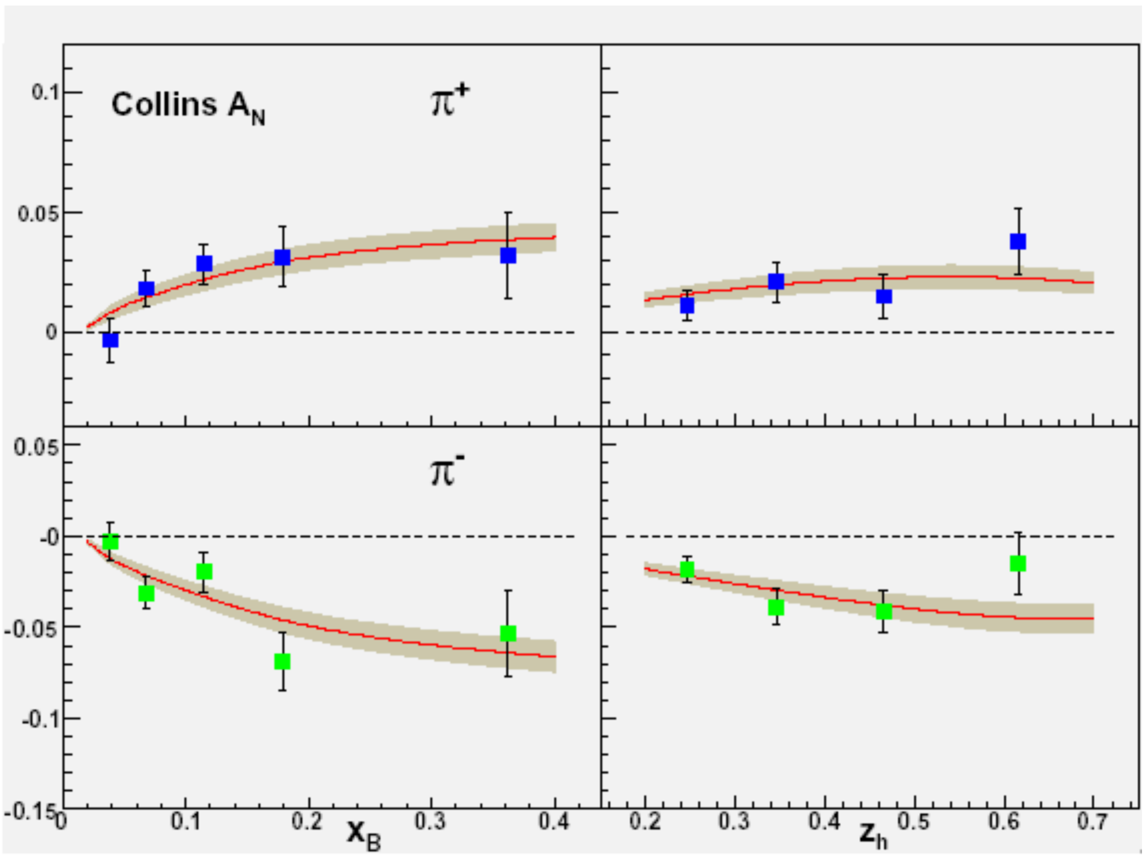
$$A_{UT}^{\sin(\Phi_h+\Phi_S)} \equiv 2\,\frac{\int \mathrm{d}\Phi_h\,\mathrm{d}\Phi_S\,[\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}]\,\sin(\Phi_h+\Phi_S)}{\int \mathrm{d}\Phi_h\,\mathrm{d}\Phi_S\,[\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}]}$$

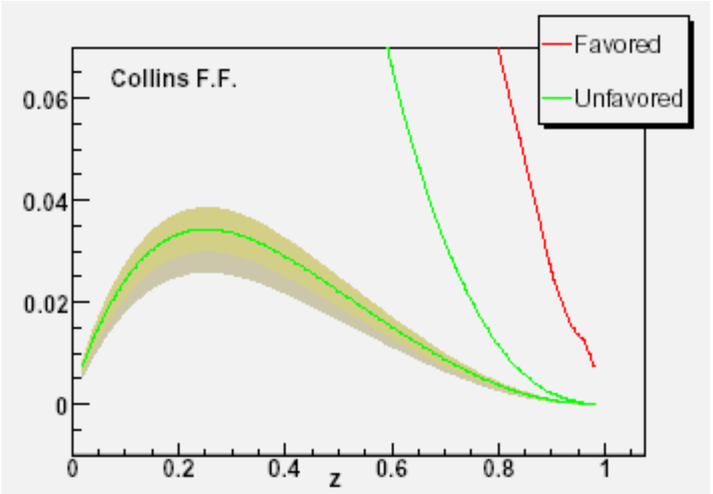
$$A_{UT}^{\sin(\Phi_h+\Phi_S)} = \frac{\sum_q \int d\Phi_S\, d\Phi_h\, d^2\boldsymbol{k}_\perp\, h_{1q}(x, k_\perp)\, \frac{d\Delta\hat{\sigma}^{\ell q\to\ell q}}{dQ^2}\, \Delta^N D_{h/q^\uparrow}(z, p_\perp) \sin(\Phi_h+\Phi_S)}{\sum_q \int d\phi_S\, d\phi_h\, d^2\boldsymbol{k}_\perp\, f_{q/p}(x, k_\perp)\, \frac{d\hat{\sigma}^{\ell q\to\ell q}}{dQ^2}\, D_{h/q}(z, p_\perp)}$$

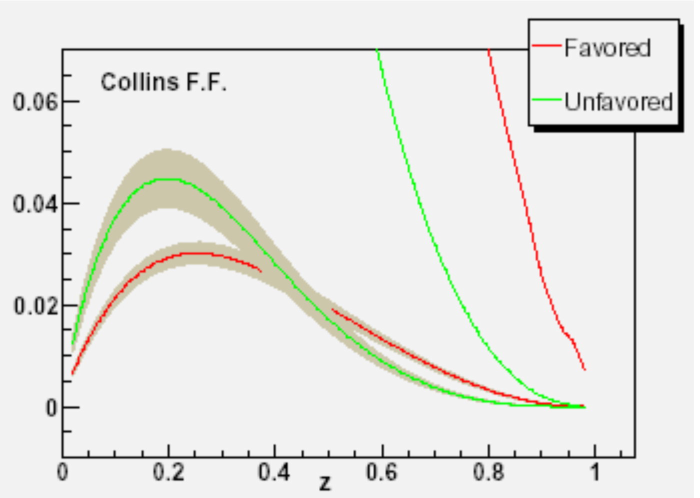
$$\mathrm{d}\Delta\hat{\sigma} = \mathrm{d}\hat{\sigma}^{\ell q^\uparrow\to\ell q^\uparrow} - \mathrm{d}\hat{\sigma}^{\ell q^\uparrow\to\ell q^\downarrow}$$

Collins effect in SIDIS couples to transversity fit to HERMES data on $A_{UT}^{\sin(\Phi_h+\Phi_S)}$

W. Vogelsang and F. Yuan



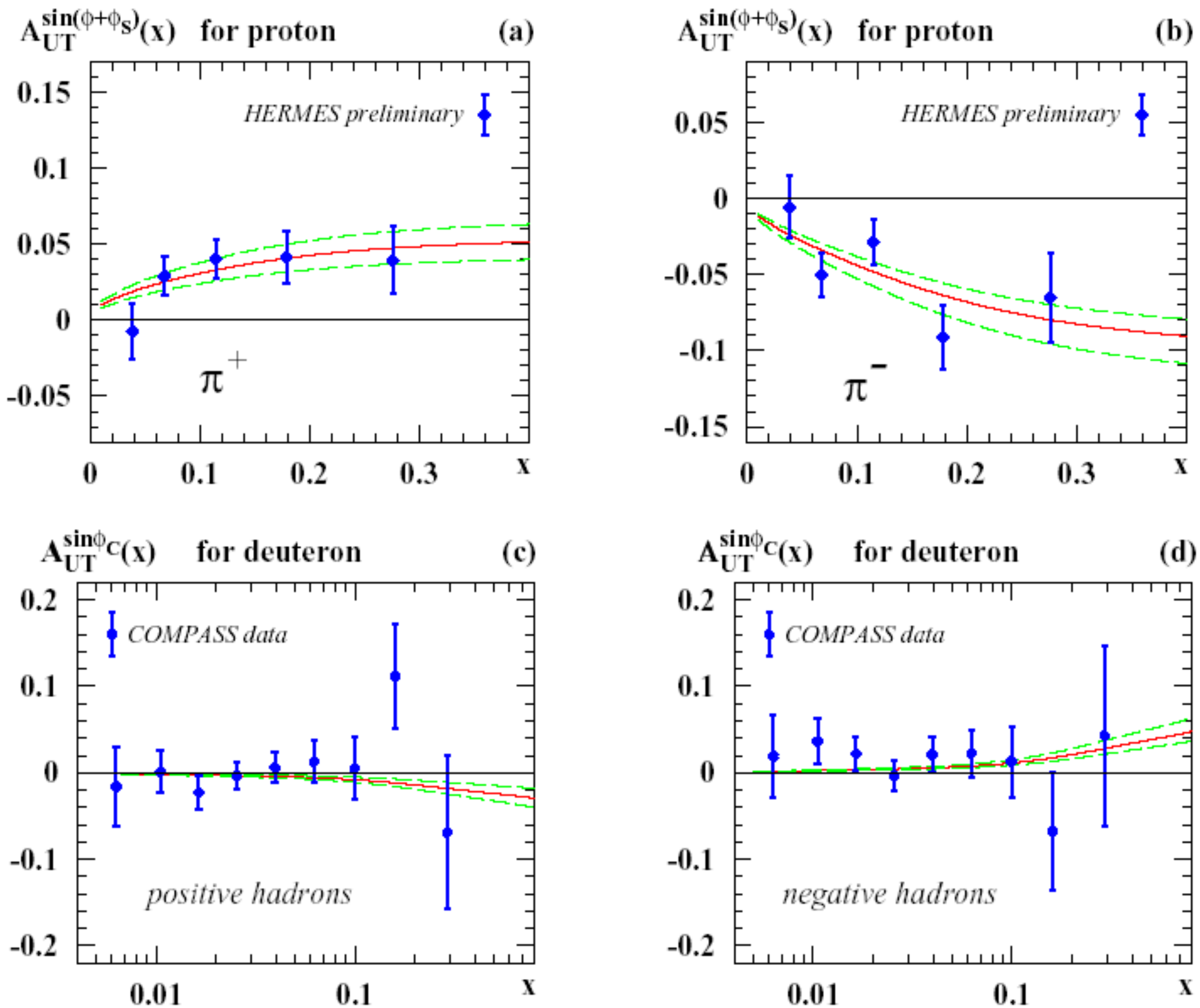




Soffer-saturated h_1 $\left(2\,|\,h_1\,| = \Delta q + q\right)$

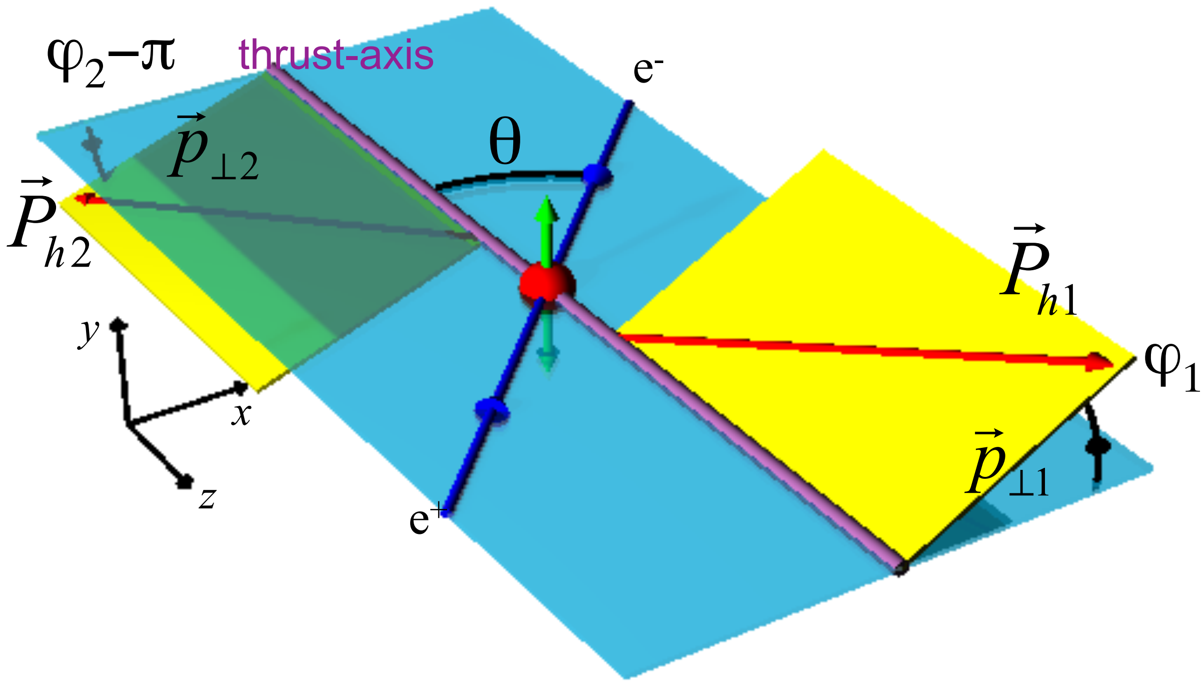
A. V. Efremov, K. Goeke and P. Schweitzer

(h_1 from quark-soliton model)



Collins function from e^+e^- processes

(spin effects without polarization, D. Boer)

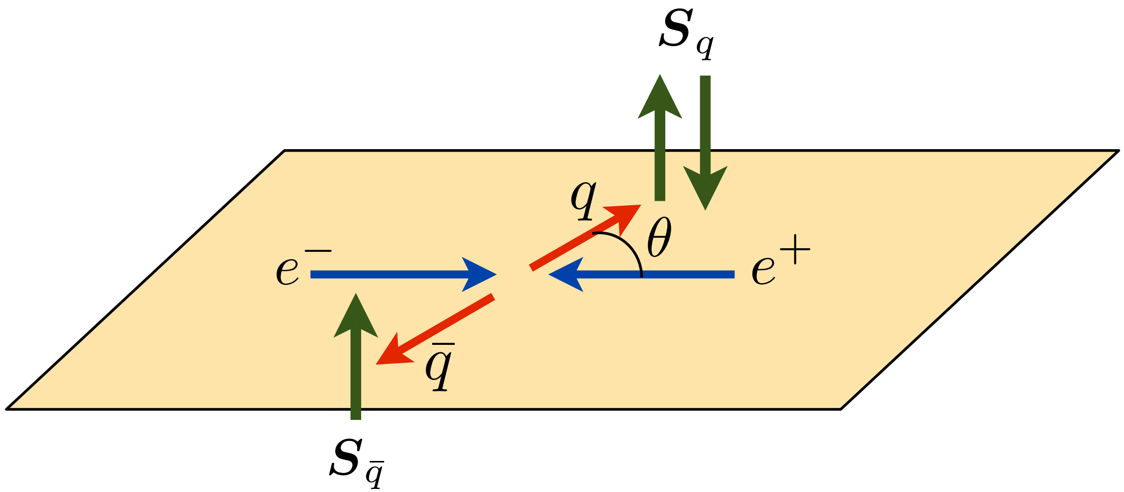


$$e^+e^- \to q\,\bar{q} \to h_1 h_2\, X$$

e+e- CMS frame:

BELLE @ KEK

$$z = \frac{2E_h}{\sqrt{s}}, \quad \sqrt{s} = 10.52\,\mathrm{GeV}$$

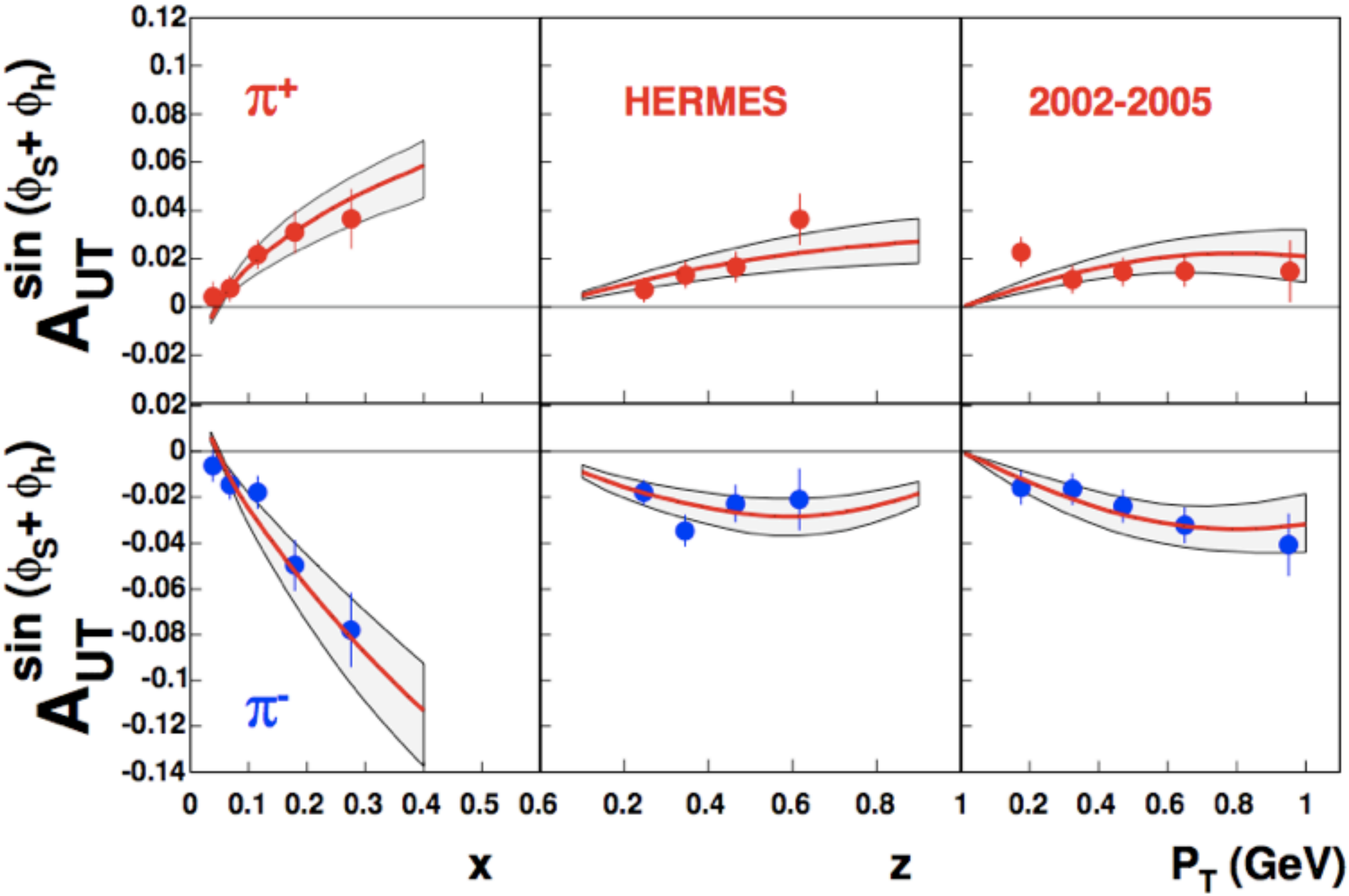


$$\frac{d\sigma^{e^+e^-\to q^\uparrow \bar{q}^\uparrow}}{d\cos\theta} = \frac{3\pi\alpha^2}{4s}\, e_q^2\, \cos^2\theta \qquad\qquad \frac{d\sigma^{e^+e^-\to q^\downarrow \bar{q}^\uparrow}}{d\cos\theta} = \frac{3\pi\alpha^2}{4s}\, e_q^2$$

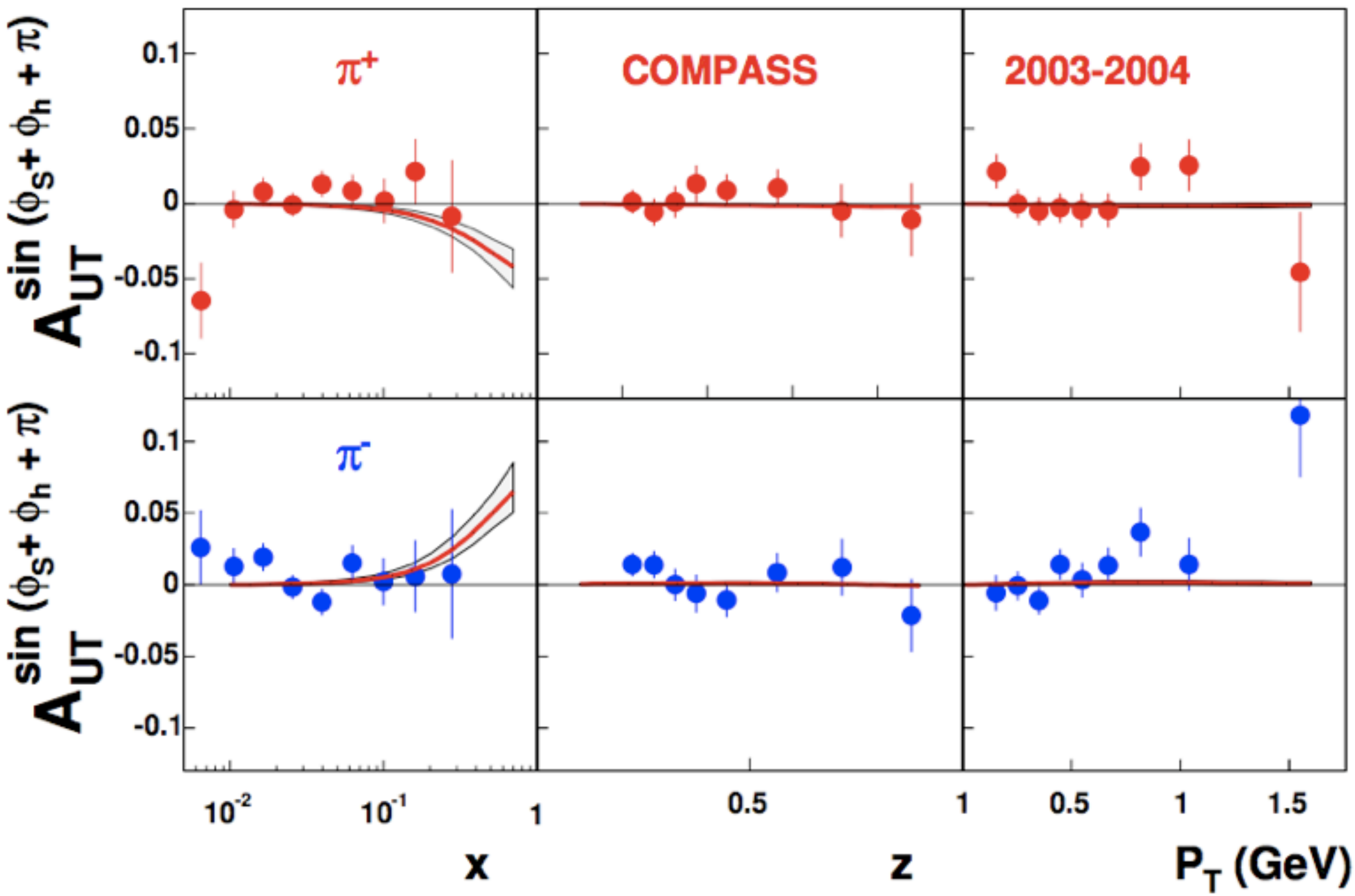
$$\begin{aligned}\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{dz_1\, dz_2\, d^2\boldsymbol{p}_{\perp 1}\, d^2\boldsymbol{p}_{\perp 2}\, d\cos\theta} &= \frac{3\pi\alpha_s^2}{2s}\sum_q e_q^2 \Big\{(1+\cos^2\theta)\, D_{h_1/q}(z_1, p_{\perp 1})\, D_{h_2/\bar{q}}(z_2, p_{\perp 2}) \\ &+ \frac{1}{4}\sin^2\theta\, \Delta^N D_{h_1/q^\uparrow}(z_1, p_{\perp 1})\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2, p_{\perp 2}) \cos(\varphi_1 {+}_2)\Big\}\end{aligned}$$

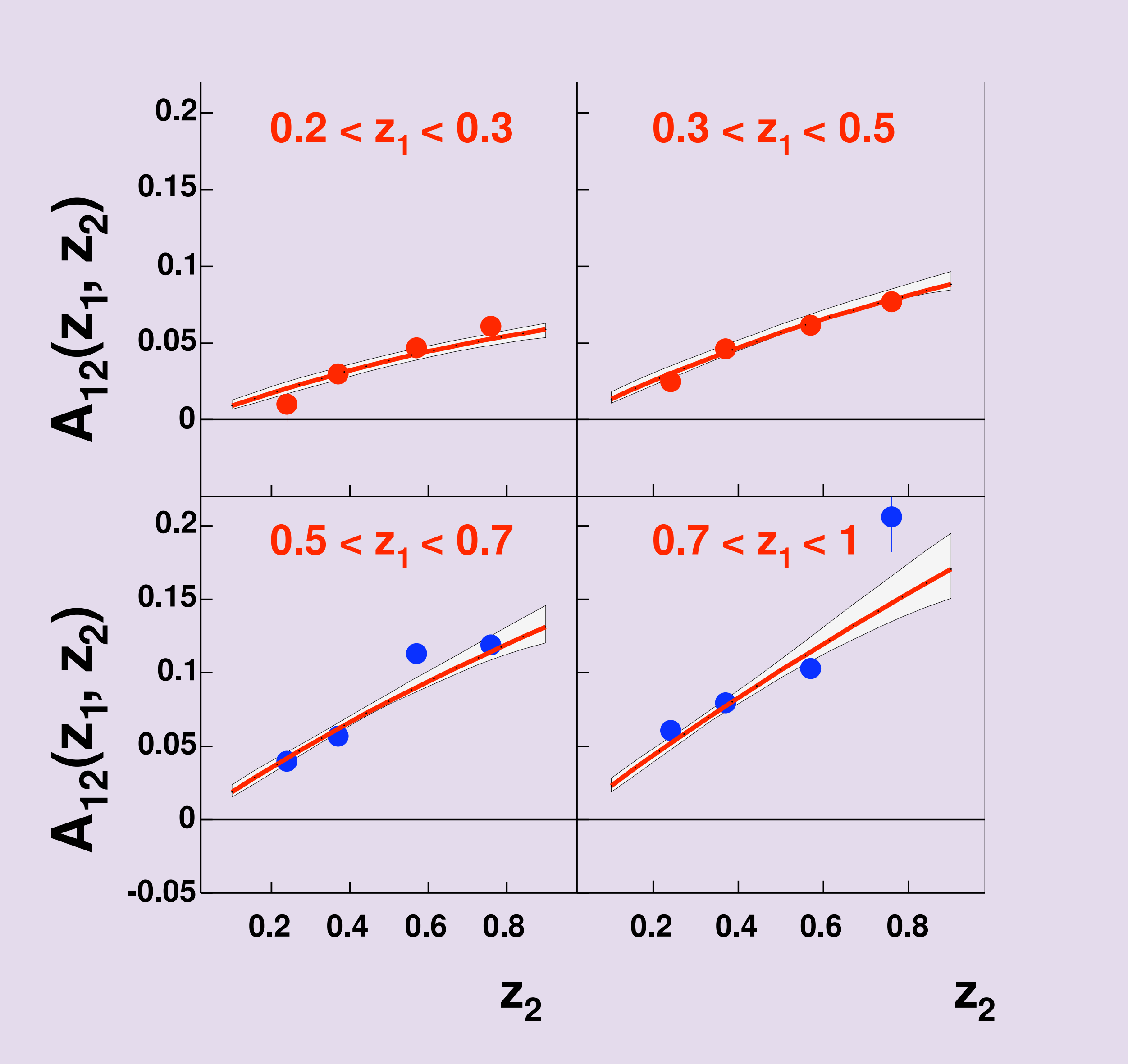
$$\begin{aligned}\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{dz_1\, dz_2\, d\cos\theta\, d\cos(\varphi_1+\varphi_2)} &= \frac{3\alpha_s^2}{4s}\sum_q e_q^2 \Big\{(1+\cos^2\theta)\, D_{h_1/q}(z_1, p_{\perp 1})\, D_{h_2/\bar{q}}(z_2, p_{\perp 2}) \\ &+ \frac{1}{4}\sin^2\theta\, \boxed{\Delta^N D_{h_1/q^\uparrow}(z_1)\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)} \cos(\varphi_1 {+}_2)\Big\}\end{aligned}$$

Collins asymmetry best fit



M. A., M. Boglione, U. D'Alesio, A. Kotzinian, S. Melis, F. Murgia,A. Prokudin and C. Türk

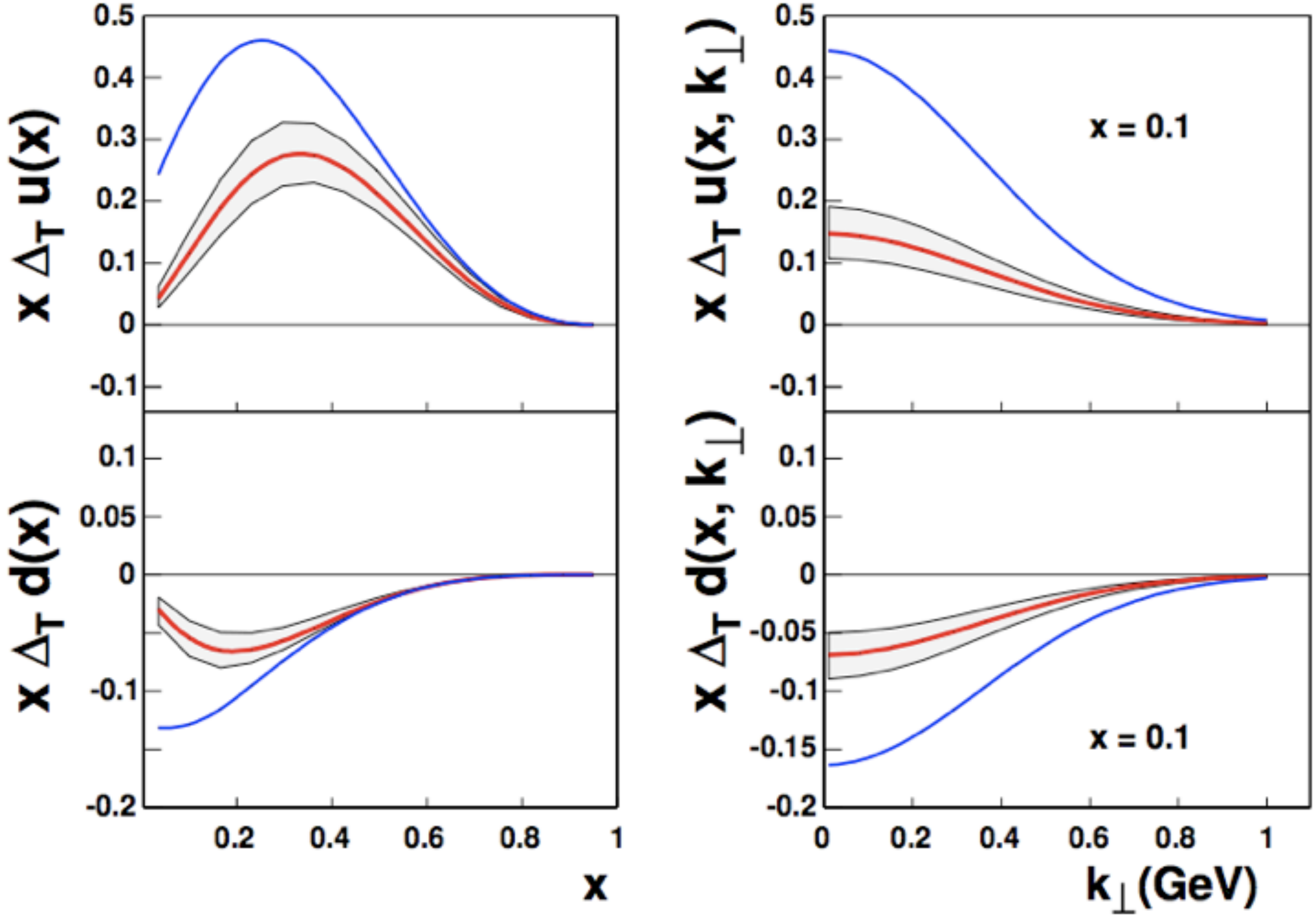




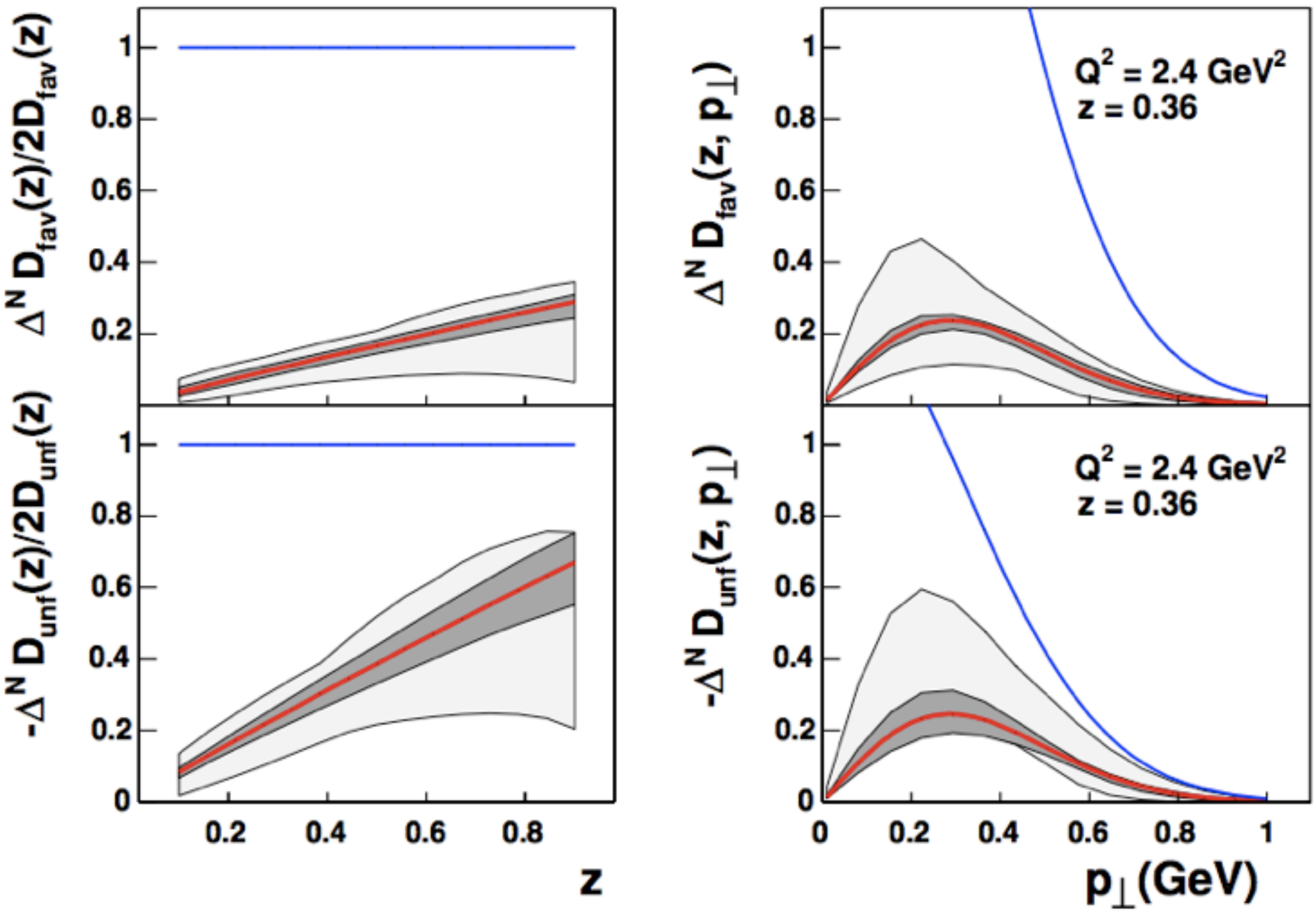
best fit of
Belle data

extracted transversity distributions

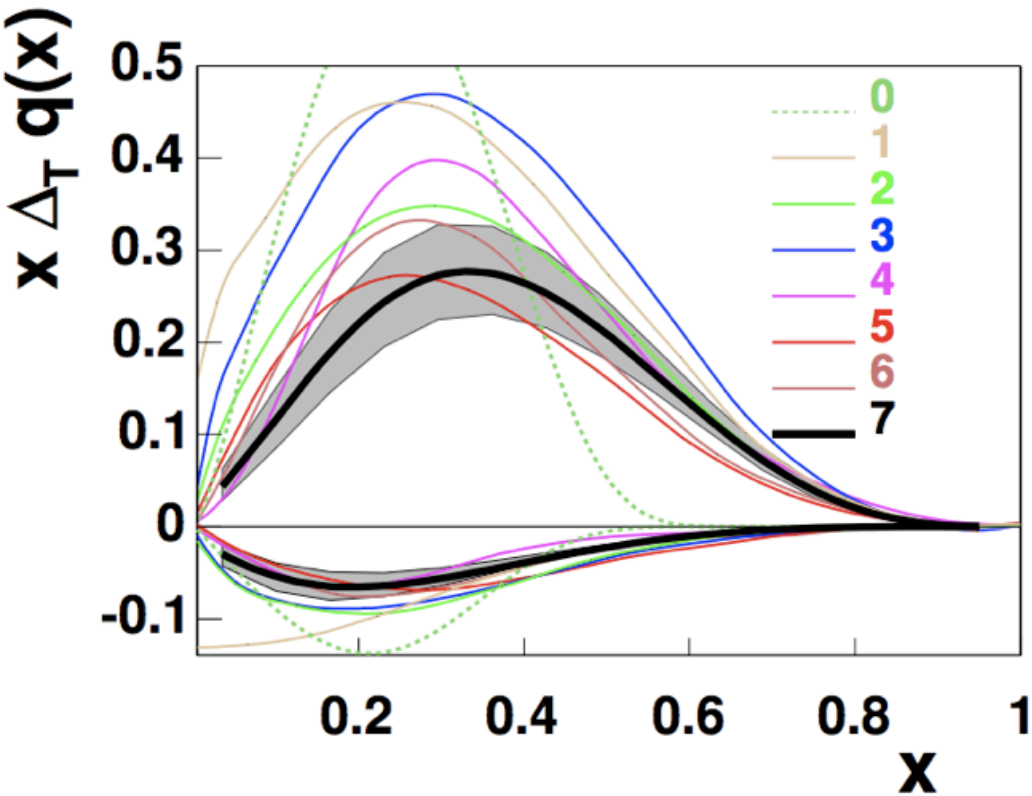
(blue lines = Soffer's bound)



extracted Collins functions

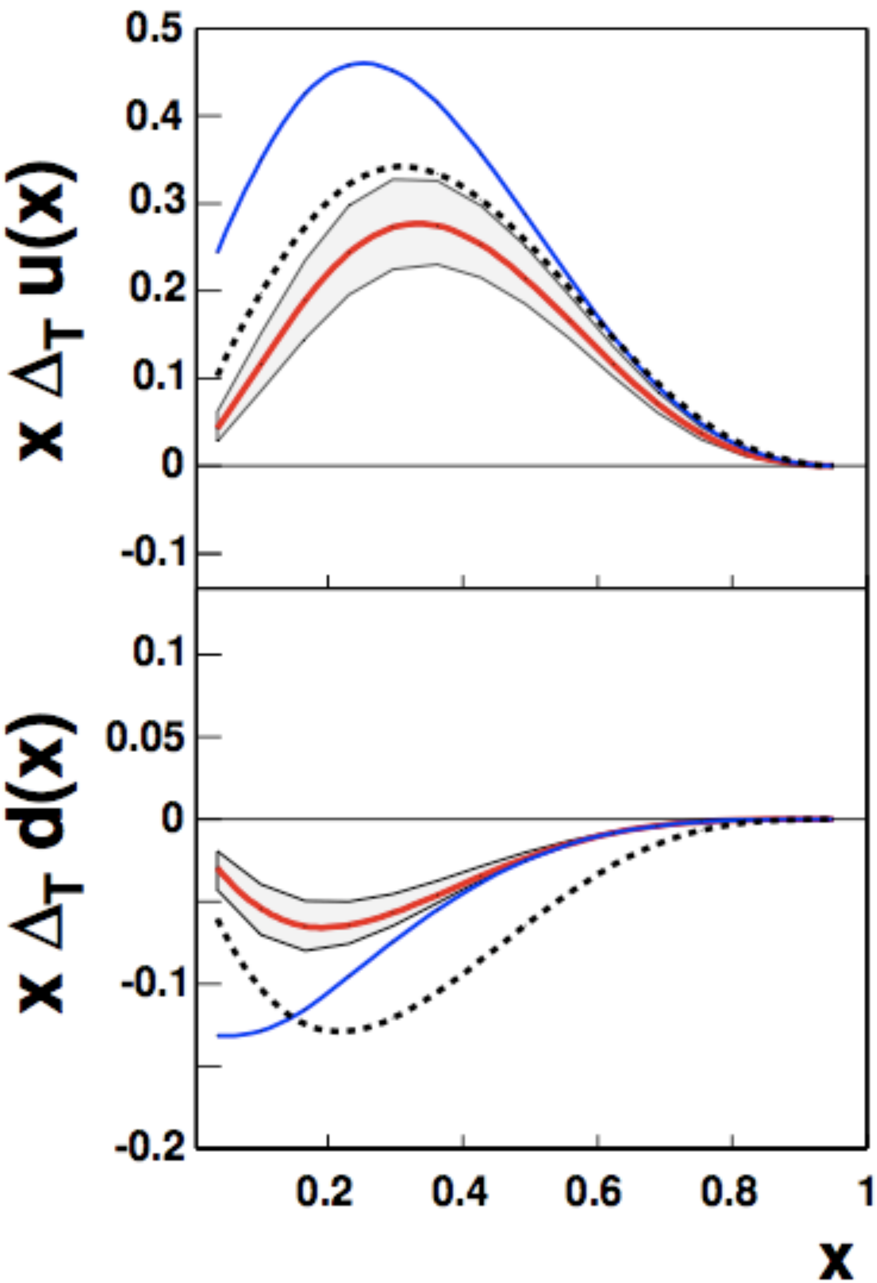


comparison of extracted transversity distributions with models



0. Barone, Calarco, Drago PLB 390 287 (97)
1. Soffer et al. PRD 65 (02)
2. Korotkov et al. EPJC 18 (01)
3. Schweitzer et al. PRD 64 (01)
4. Wakamatsu, PLB B653 (07)
5. Pasquini et al., PRD 72 (05)
6. Cloet, Bentz and Thomas PLB 659 (08)
7. This analysis.

transversity vs. helicity



1. Solid red line – transversity distribution

$$\Delta_T q(x)$$

this analysis at $Q^2 = 2.4\ \mathrm{GeV}^2$

2. Solid blue line – Soffer bound

$$\frac{q(x) + \Delta q(x)}{2}$$

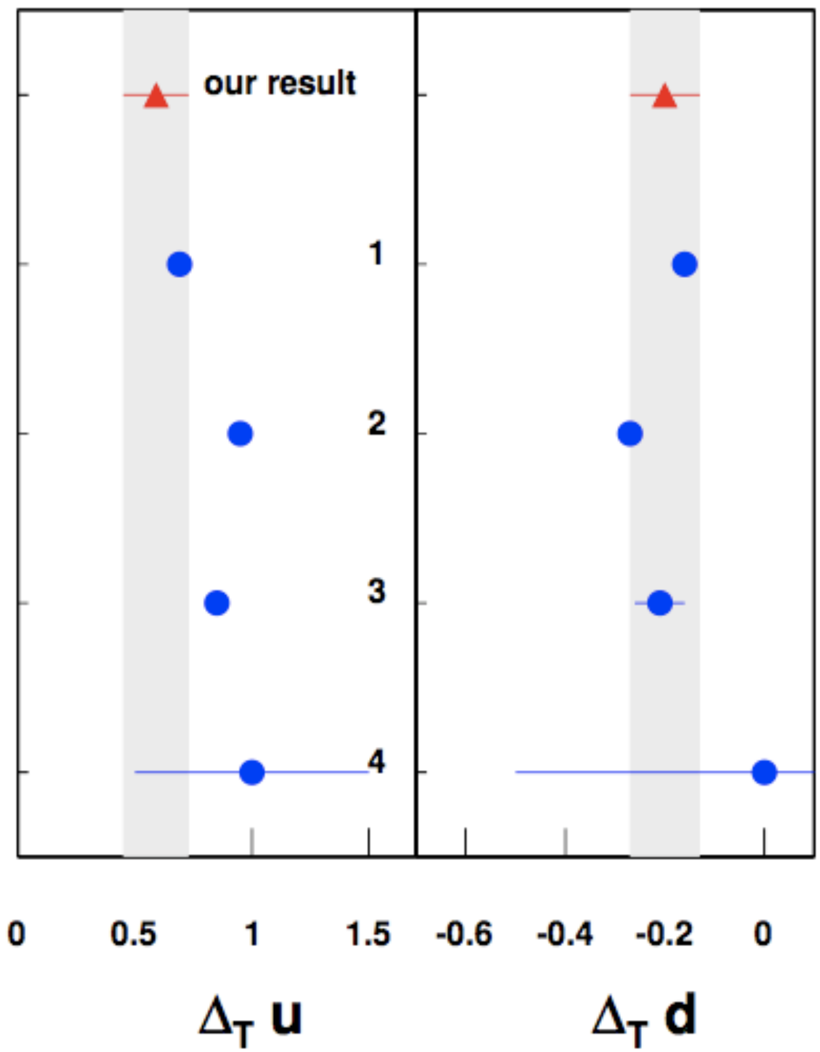
GRV98LO + GRSV98LO

3. Dashed line – helicity distribution

$$\Delta q(x)$$

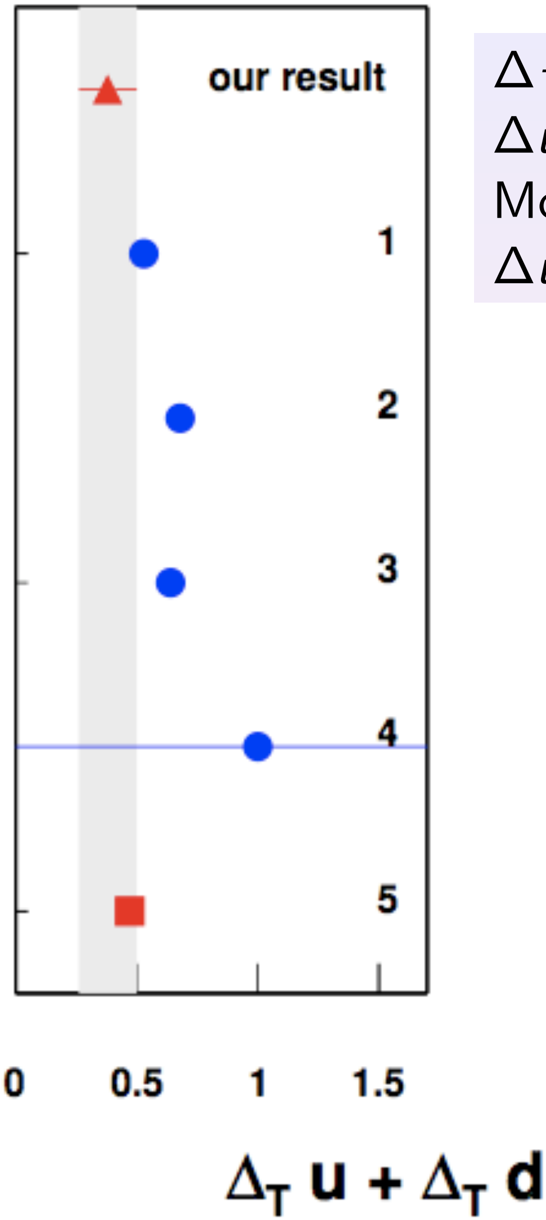
Tensor charges $(\Delta_T \bar{q} = 0)$

$$\Delta_T u = 0.59^{+0.14}_{-0.13}, \ \Delta_T d = -0.20^{+0.05}_{-0.07} \text{ at } Q^2 = 0.8 \text{ GeV}^2$$



1. Quark-diquark model:
   Cloet, Bentz and Thomas
   PLB **659**, 214 (2008), $Q^2 = 0.4$ GeV^2
2. CQSM:
   M. Wakamatsu, PLB B **653** (2007) 398
   $Q^2 = 0.3$ GeV^2
3. Lattice QCD:
   M. Gockeler et al.,
   Phys.Lett.B627:113-123,2005 , Q^2 =
   GeV^2
4. QCD sum rules:
   Han-xin He, Xiang-Dong Ji,
   PRD 52:2960-2963,1995, $Q^2 \sim 1$ GeV^2

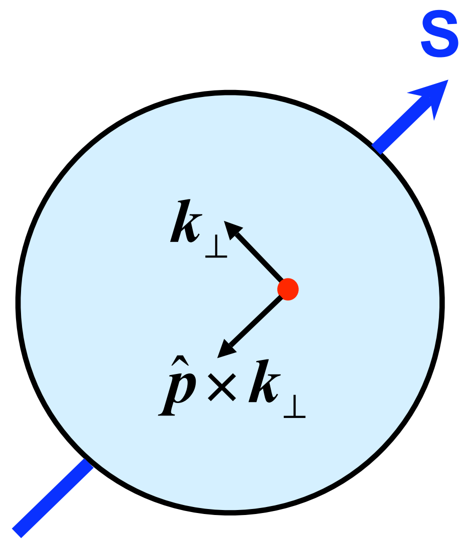
Transversity vs. helicity



$\Delta_T u = 0.59^{+0.14}_{-0.13}$, $\Delta_T d = -0.20^{+0.05}_{-0.07}$ at $Q^2 = 0.8$ GeV^2
$\Delta u = 0.87$, $\Delta d = -0.39$ at $Q^2 = 0.8$ GeV^2
More informative, the contribution to the spin:
$\Delta u + \Delta d = 0.47$, $\Delta_T u + \Delta_T d = 0.38^{+0.12}_{-0.08}$

1. Quark-diquark model:
   Cloet, Bentz and Thomas
   PLB **659**, 214 (2008), $Q^2 = 0.4$ GeV^2
2. CQSM:
   M. Wakamatsu, PLB B **653** (2007) 398.
   $Q^2 = 0.3$ GeV^2
3. Lattice QCD:
   M. Gockeler et al.,
   Phys.Lett.B627:113-123,2005 , $Q^2 = 4$ GeV^2
4. QCD sum rules:
   Han-xin He, Xiang-Dong Ji,
   PRD 52:2960-2963,1995, $Q^2 \sim 1$ GeV^2
5. $\Delta u + \Delta d = 0.47$

What do we learn from the Sivers distribution?



number density of partons with longitudinal momentum fraction x and transverse momentum $\boldsymbol{k}_\perp$, inside a proton with spin $\mathbf{S}$

$$\boxed{\sum_a \int dx\, d^2\boldsymbol{k}_\perp\, \boldsymbol{k}_\perp\, f_{a/p^\uparrow}(x, \boldsymbol{k}_\perp) \equiv \sum_a \langle \boldsymbol{k}_\perp^a \rangle = 0}$$

M. Burkardt, PR **D69**, 091501 (2004)

Total amount of intrinsic momentum carried by partons of flavour a

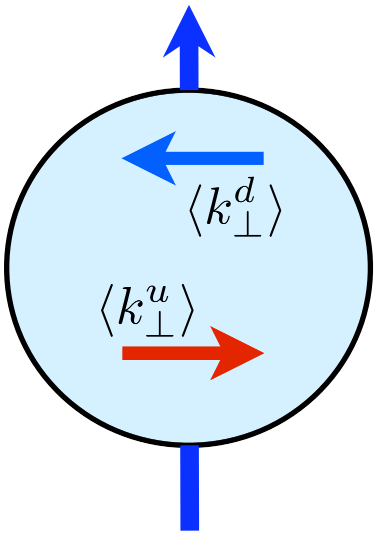
$$\langle \boldsymbol{k}_\perp^a \rangle = \left[\frac{\pi}{2} \int_0^1 dx \int_0^\infty dk_\perp \, k_\perp^2 \, \Delta^N f_{a/p^\uparrow}(x, k_\perp) \right] (\boldsymbol{S} \times \hat{\boldsymbol{P}})$$

$$= m_p \int_0^1 dx \, \Delta^N f^{(1)}_{q/p^\uparrow}(x) \, (\boldsymbol{S} \times \hat{\boldsymbol{P}}) \equiv \langle k_\perp^a \rangle \, (\boldsymbol{S} \times \hat{\boldsymbol{P}})$$

$$\langle k_\perp^u \rangle + \langle k_\perp^d \rangle = -17^{+37}_{-55} \ (\mathrm{MeV}/c)$$

$$[\, \langle k_\perp^u \rangle = 96^{+60}_{-28} \qquad \langle k_\perp^d \rangle = -113^{+45}_{-51} \,]$$

$$\langle k_\perp^{\bar{u}} \rangle + \langle k_\perp^{\bar{d}} \rangle + \langle k_\perp^s \rangle + \langle k_\perp^{\bar{s}} \rangle = -14^{+43}_{-66} \ (\mathrm{MeV}/c)$$

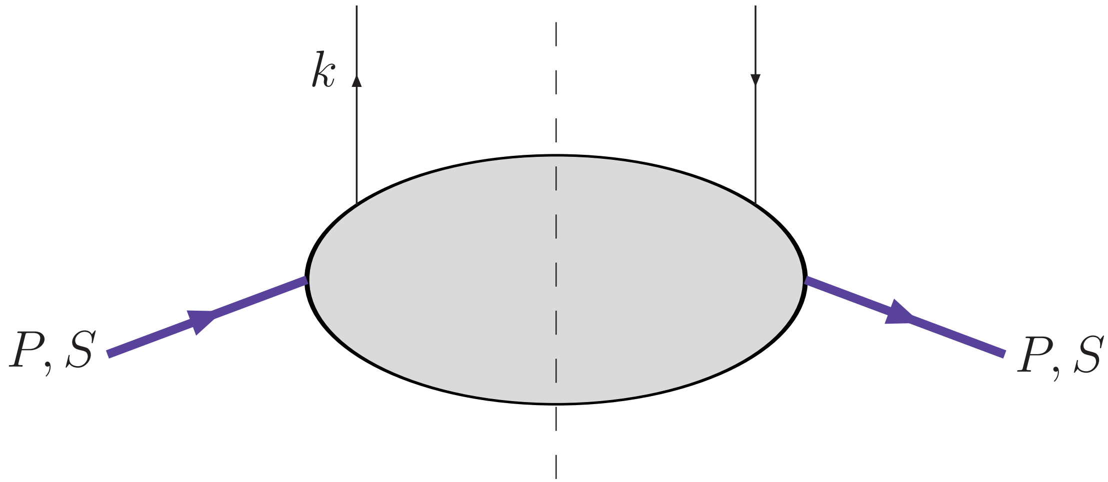


Burkardt sum rule almost saturated by u and d quarks alone; little residual contribution from gluons

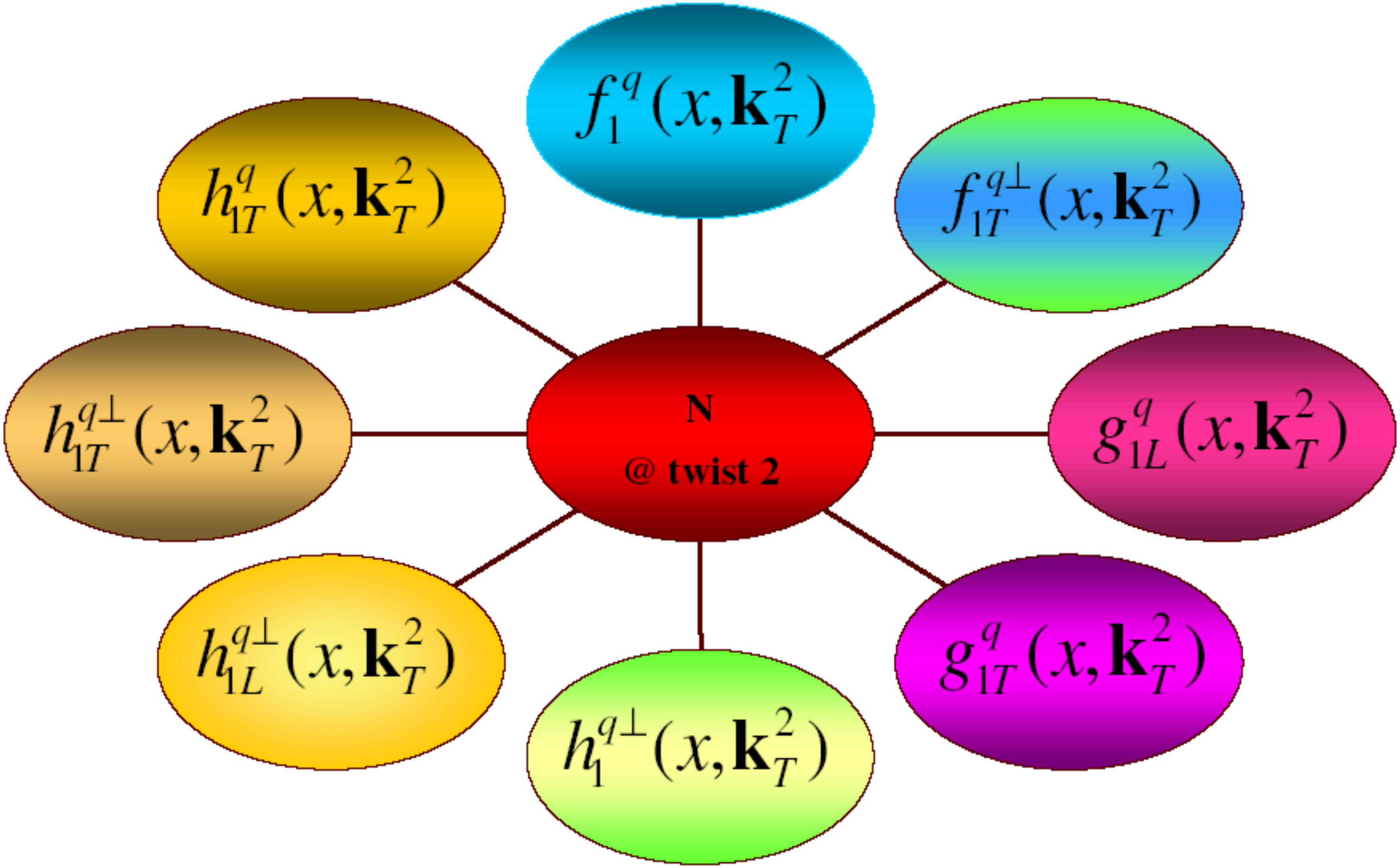
$$-10 \leq \langle k_\perp^g \rangle \leq 48 \ (\mathrm{MeV}/c)$$

The leading-twist correlator, with intrinsic $k_\perp$, contains several other functions .....

$$
\begin{aligned}
\Phi(x, \boldsymbol{k}_\perp) &= \frac{1}{2}\Big[f_1 \not{n}_+ + f_{1T}^\perp \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu k_\perp^\rho S_T^\sigma}{M} + \left(S_L\, g_{1L} + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M}\, g_{1T}^\perp \right) \gamma^5 \not{n}_+ \\
&+ h_{1T}\, i\sigma_{\mu\nu}\gamma^5 n_+^\mu S_T^\nu + \left(S_L\, h_{1L}^\perp + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M}\, h_{1T}^\perp \right) \frac{i\sigma_{\mu\nu}\gamma^5 n_+^\mu k_\perp^\nu}{M} \\
&+ h_1^\perp \frac{\sigma_{\mu\nu} k_\perp^\mu n_+^\nu}{M} \Big]
\end{aligned}
$$



8 leading-twist spin-$k_\perp$ dependent distribution functions



Courtesy of Aram Kotzinian

$$
\begin{aligned}
d\sigma \;=\;& d\sigma^{0}_{UU} + \cos 2\Phi_h\, d\sigma^{1}_{UU} + \frac{1}{Q}\,\cos\Phi_h\, d\sigma^{2}_{UU} + \lambda\,\frac{1}{Q}\,\sin\Phi_h\, d\sigma^{3}_{LU} \\
+\;& S_L \left\{ \sin 2\Phi_h\, d\sigma^{4}_{UL} + \frac{1}{Q}\,\sin\Phi_h\, d\sigma^{5}_{UL} + \lambda \left[d\sigma^{6}_{LL} + \frac{1}{Q}\,\cos\Phi_h\, d\sigma^{7}_{LL} \right] \right\} \\
+\;& S_T \left\{ \sin(\Phi_h - \Phi_S)\, d\sigma^{8}_{UT} + \sin(\Phi_h + \Phi_S)\, d\sigma^{9}_{UT} + \sin(3\Phi_h - \Phi_S)\, d\sigma^{10}_{UT} \right. \\
+\;& \frac{1}{Q}\left[\sin(2\Phi_h - \Phi_S)\, d\sigma^{11}_{UT} + \sin\Phi_S\, d\sigma^{12}_{UT} \right] \\
+\;& \left. \lambda \left[\cos(\Phi_h - \Phi_S)\, d\sigma^{13}_{LT} + \frac{1}{Q}\left(\cos\Phi_S\, d\sigma^{14}_{LT} + \cos(2\Phi_h - \Phi_S)\, d\sigma^{15}_{LT} \right) \right] \right\}
\end{aligned}
$$

SIDISLAND

Kotzinian, **NP B441** (1995) 234

Mulders and Tangermann, **NP B461** (1996) 197

Boer and Mulders, **PR D57** (1998) 5780

Bacchetta et al., **PL B595** (2004) 309

Bacchetta et al., **JHEP 0702** (2007) 093